Easy SDR Experimentation
with
GNU Radio

Introduction to DSP

(and some GNU Radio)

About Me

- EE, Independent Consultant
  - Hardware, Software, Security
  - Cellular, FPGA, GNSS, ...
- DAGR – Denver Area GNU Radio meet-up

Purpose

- Get you into SDR!
- Cover the basics of SDR to get you started
- Not – Comprehensive coverage
- Not – “How To Do X”
- Some examples to make it real

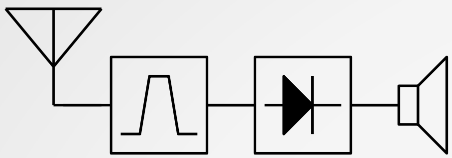
Audience

- Well, you... duh!
- Radio fundamentals
- Algebra, Trigonometry (just a little!)

Software Defined Radio (SDR)

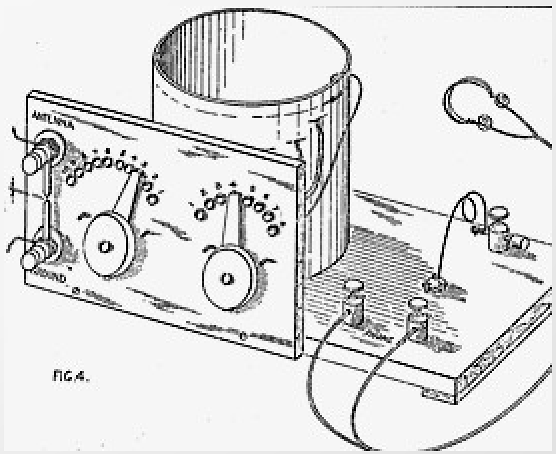
- What is it?
- “Software”?
  - Implies generalized hardware, reconfigurable for a specific purpose
  - E.g. Computer / Software
- But really… “Digital”
  - Convert an analog signal to digital data
  - Process in the digital realm, rather than analog
    - Digital Signal Processing (DSP)

Analog Receiver Evolution

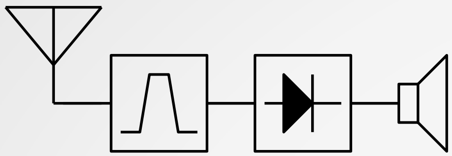


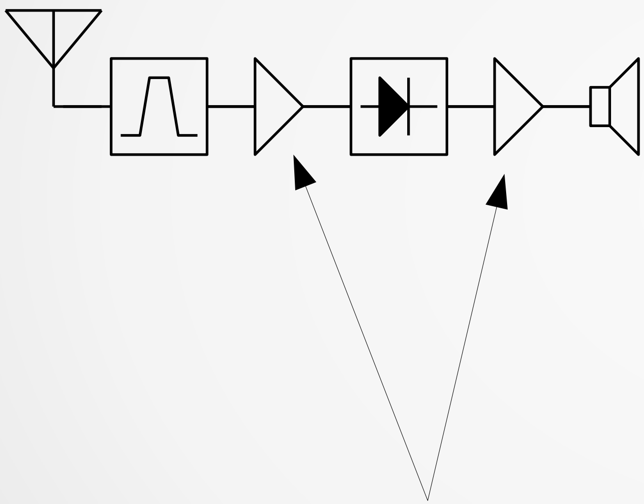
Detector

Frequency Selectivity

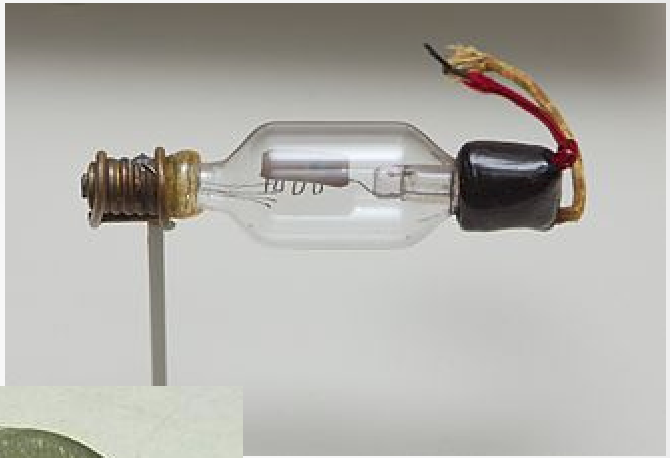


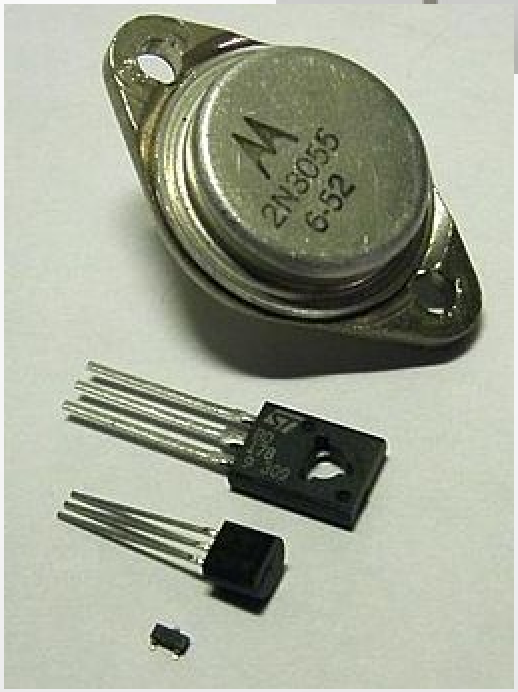
Analog Receiver Evolution



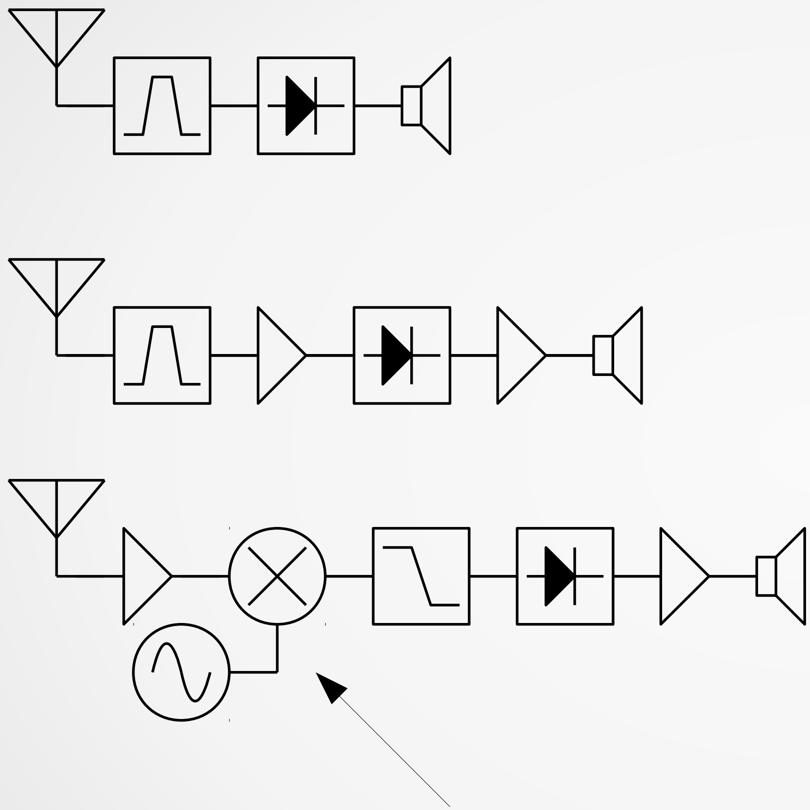


Gain!



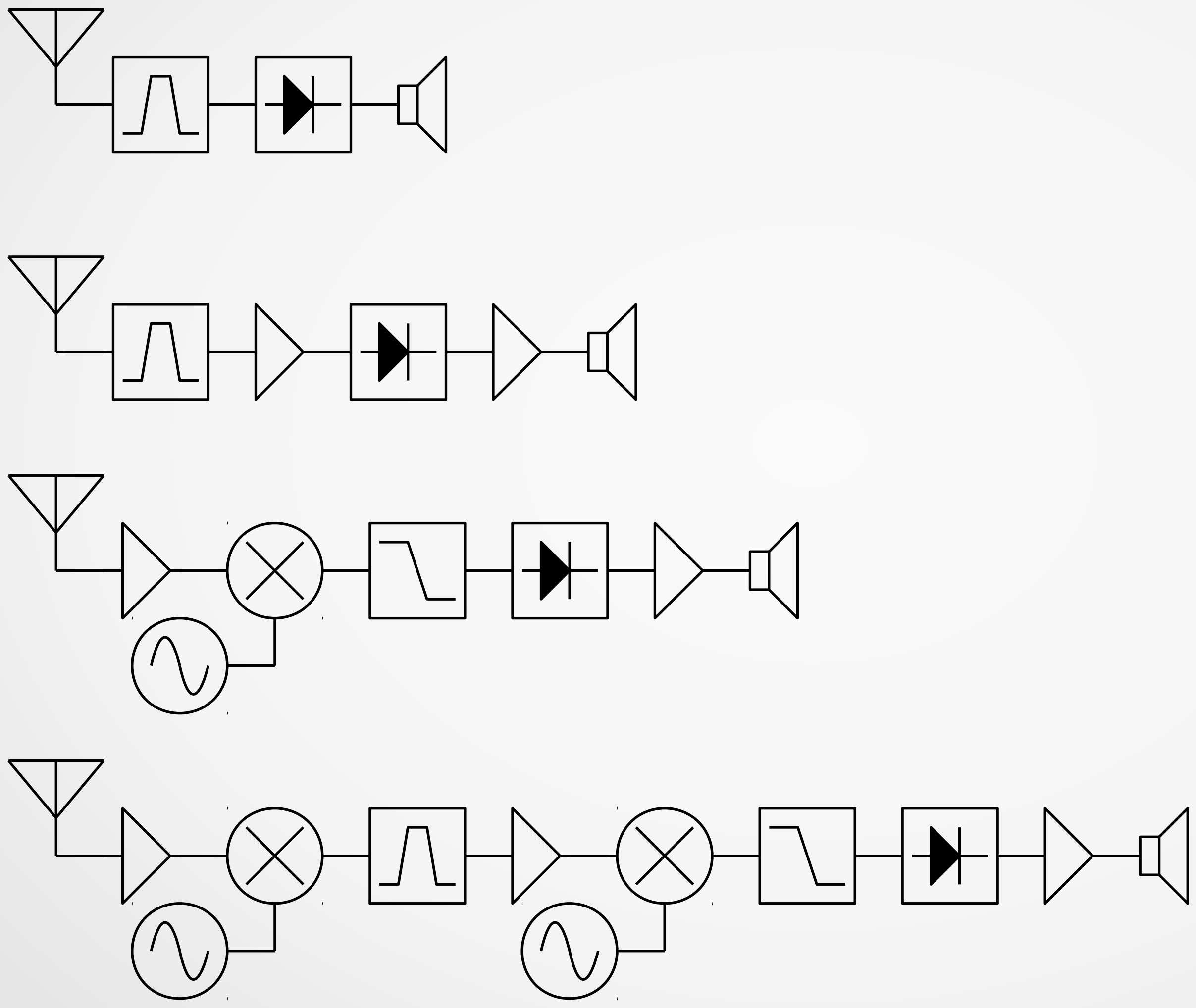


Analog Receiver Evolution

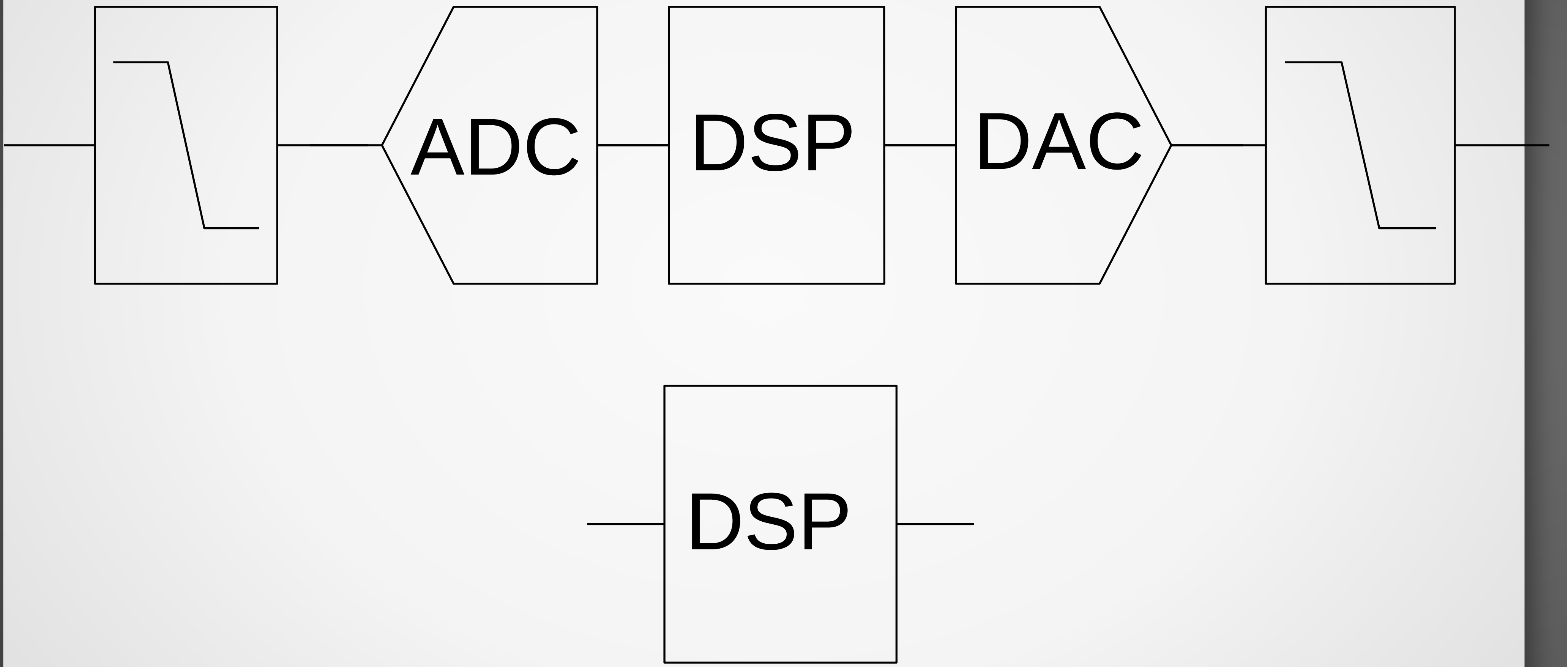


Frequency Conversion

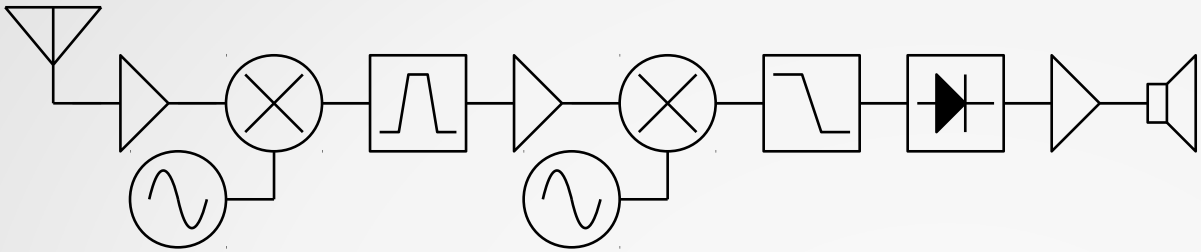
Analog Receiver Evolution

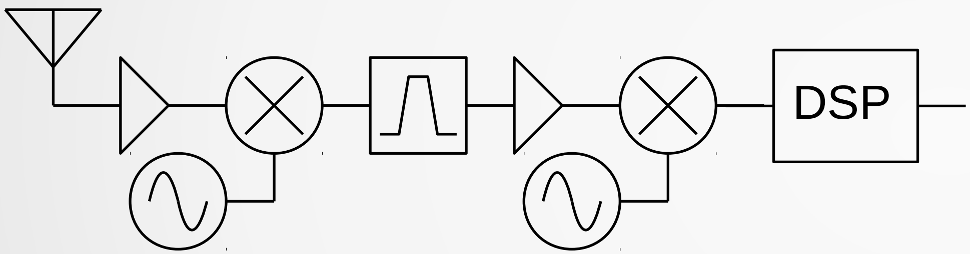


Digital Conversion



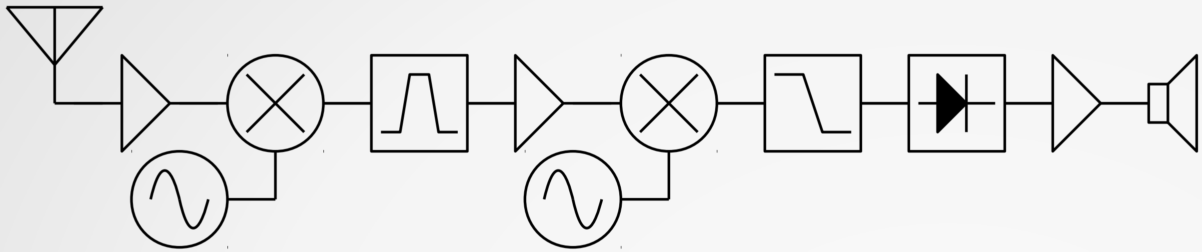
SDR Evolution

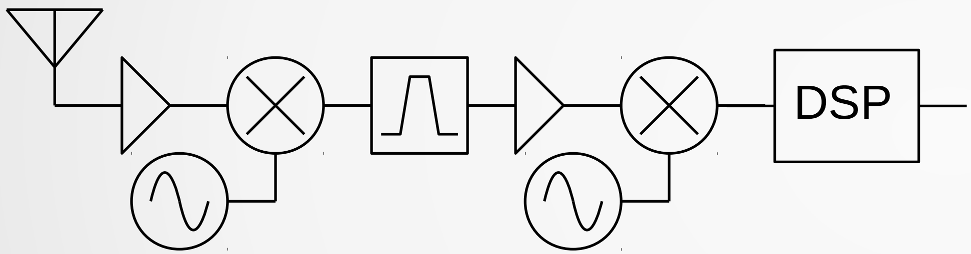


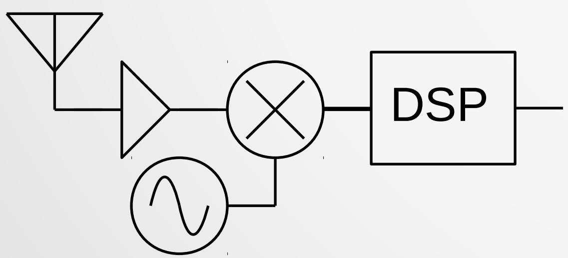


- Super-Heterodyne
  - Demod filtering, processing
  - Detector/Demodulator
  - Additional final IF filtering

SDR Evolution

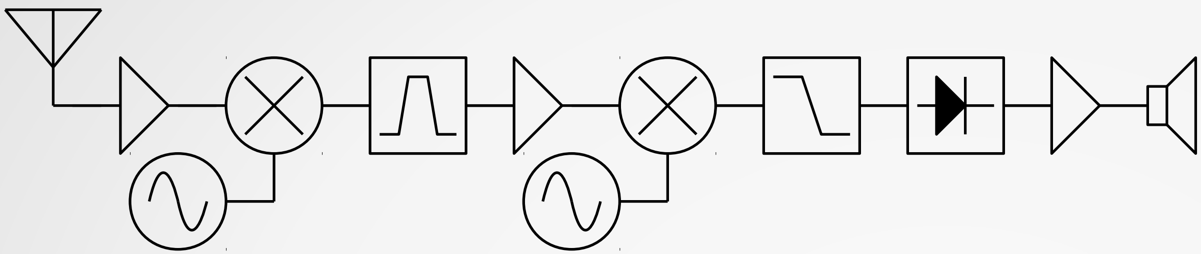


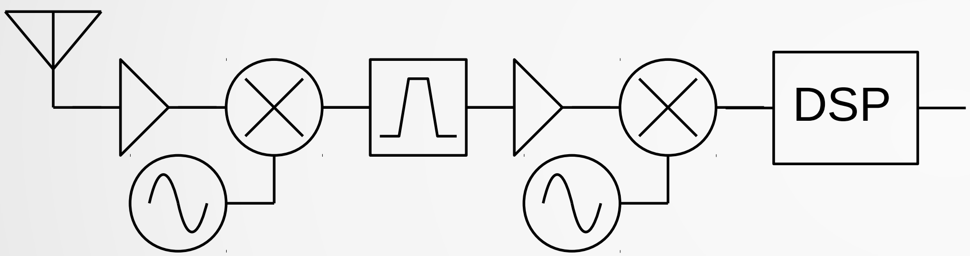


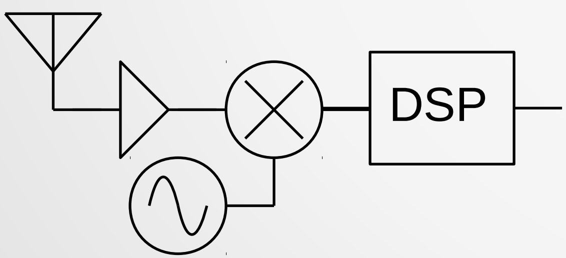


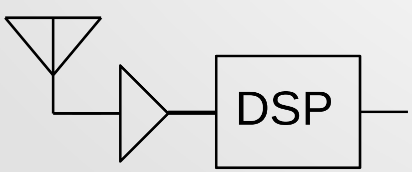
- Sub-sampling
- Zero-IF / Direct-Conversion
  - I/Q, Quadrature

SDR Evolution









- Direct Sampling, the final frontier

Why?

- Flexibility!
- Avoid analog component imperfections
  - Tolerances, Non-linearity, etc.
  - The math doesn’t change
- Greater performance – e.g. very sharp filters
- Sometimes cost
- Things you just wouldn’t do in analog (OFDM)
- Moore’s Law...

Why for Amateurs?

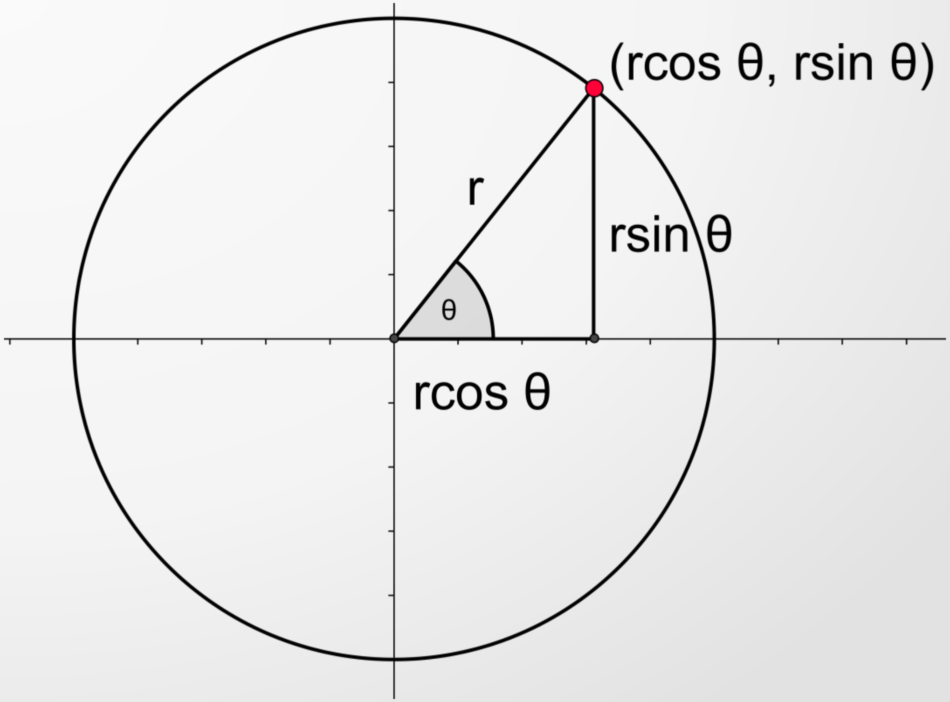
- Flexibility
- Performance
- Advanced modulations
- Digital modes
- Experimentation!

Basic DSP Concepts

- Signals
- Time ↔ Frequency Domains
- Filtering
- Sampling
- Sample Rate Conversion
- I/Q, Quadrature, Analytic Signals
- Frequency Conversion
- De/modulation

Signal

- Sinusoids
- Unit Circle, Trigonometry
- sin(θ)=opp/hyp, cos(θ)=adj/hyp
- If r=hyp=1 and adj=x, opp=y
  - y=sin(θ), x=cos(θ)
- Frequency is speed around circle
  - Hz (cycles/sec) = 2*pi (rad/sec)



Time ↔ Frequency Domains

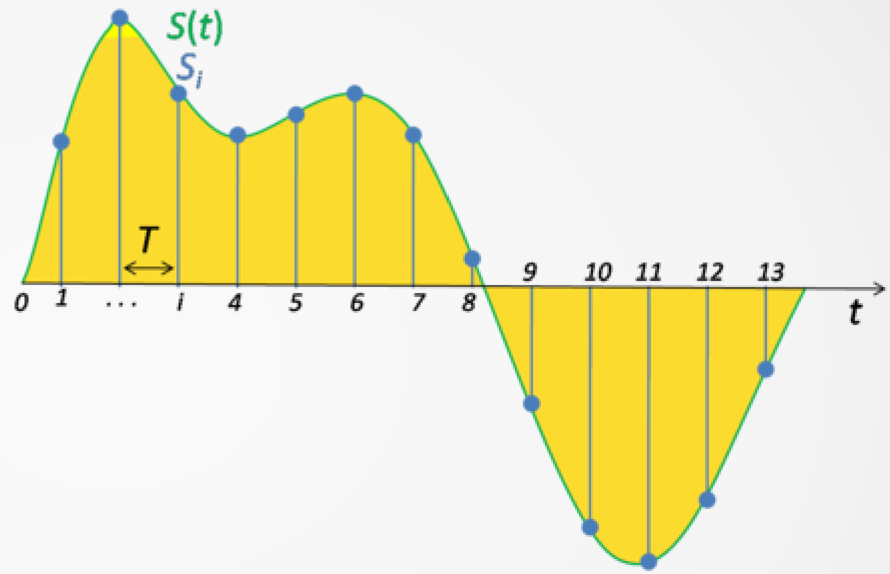
- Different ways of looking at a signal
- Transforms, Fourier, DFT/FFT
- Sine wave →”spike”
- Square wave → Odd harmonics
- Pulse → Sinc... sin(x)/x
- Negative frequency

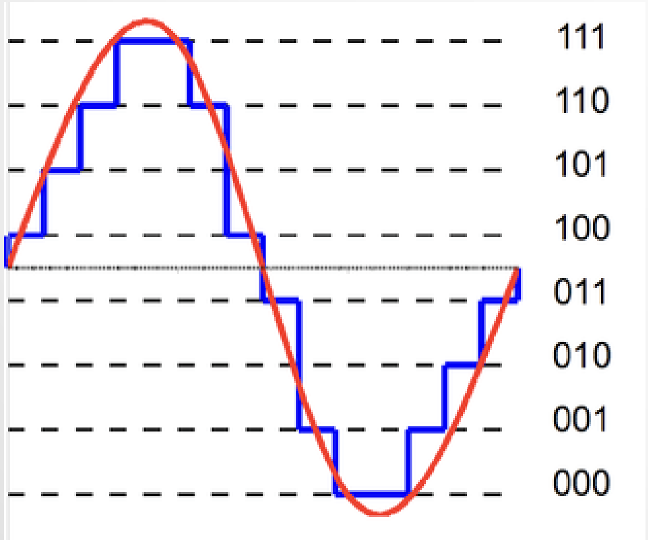
Filtering

- Change frequency response
  - and/or phase
- Filtering = convolution
- Convolution and multiplication are time-frequency pairs
- FIR/IIR

Sampling

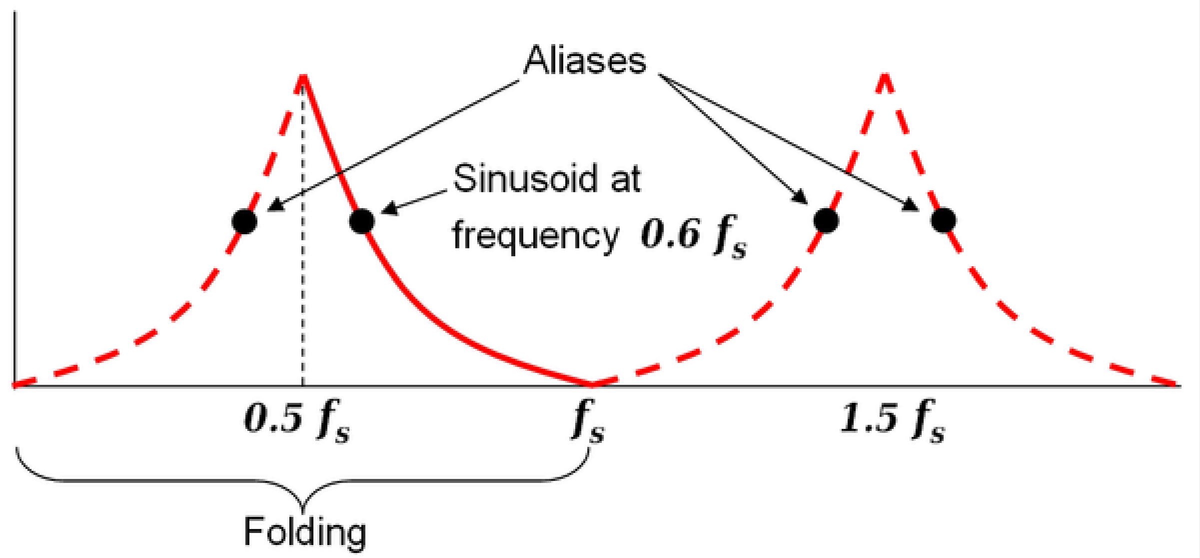
- Sampling
  - Discrete time
- Quantization
  - Discrete value





Sampling

- Nyquist frequency (½ f_s)
- Spectral Folding
  - Aliasing
  - Inversion
  - Sub-sampling



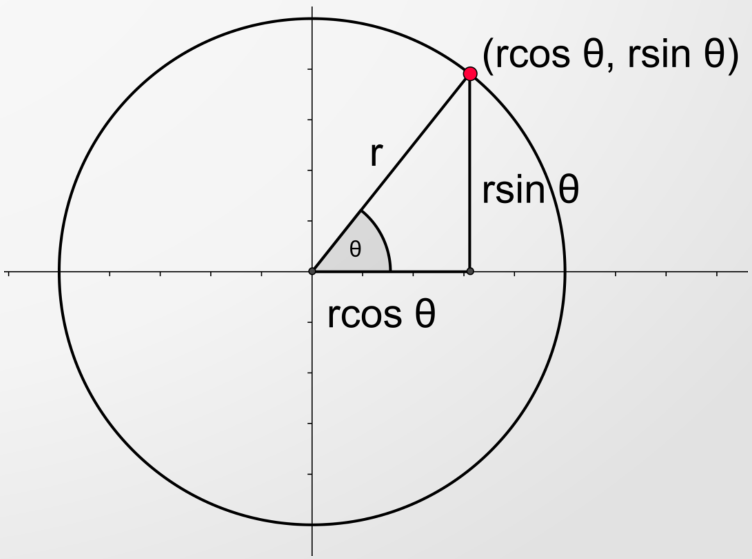
Sample Rate Conversion

- Decimation
- Interpolation
- Aliasing / Filtering

(Pause before I/Q)

I/Q Sampling

- **I**n-phase and **Q**uadrature-phase
- AKA Quadrature, Analytic Signal
- Complex Numbers
- VERY common in DSP/SDR
- Very common area of newcomer confusion



SSB $\rightarrow$ I/Q

- Single sideband modulation and IQ sampling are very similar
- Use SSB to understand IQ

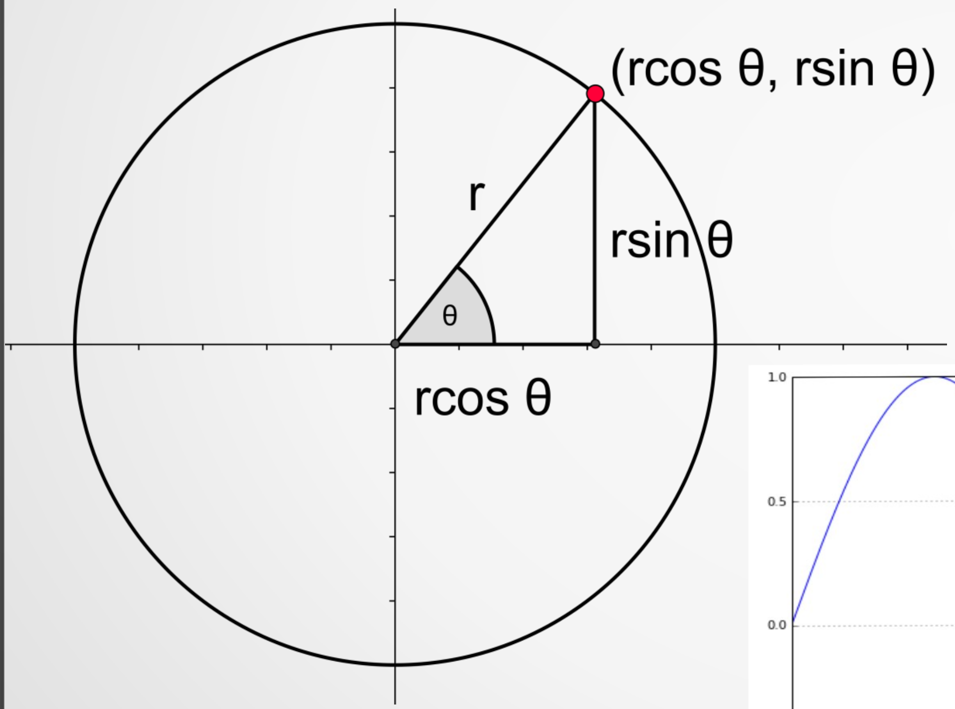
What is SSB?

- A derivative of Amplitude Modulation (AM)
- To understand SSB, first understand AM
- Before AM, understand “modulation”

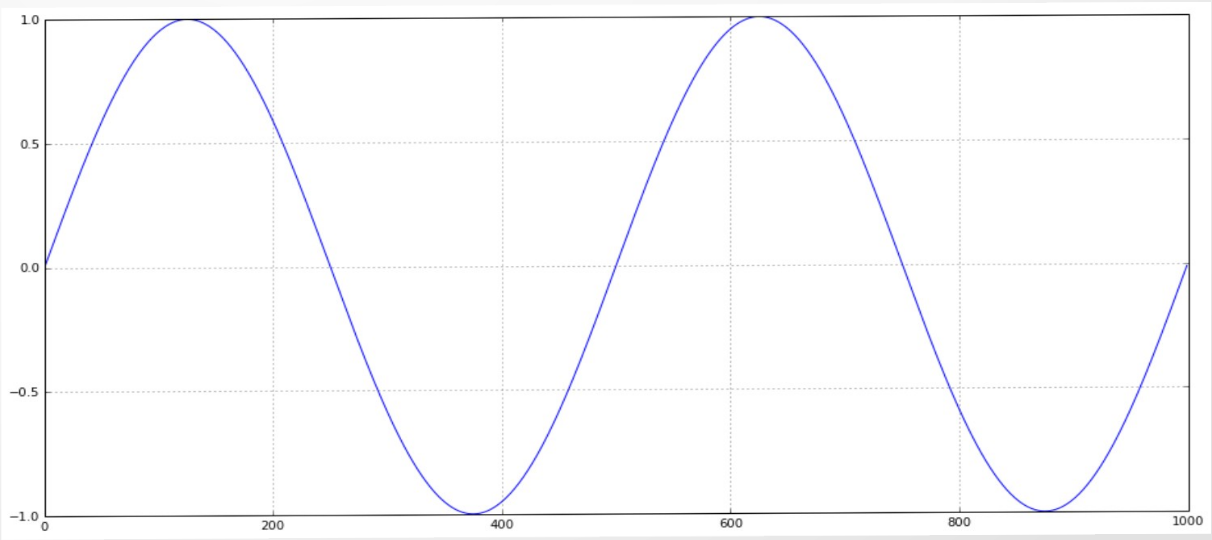
Here we go...

Modulation

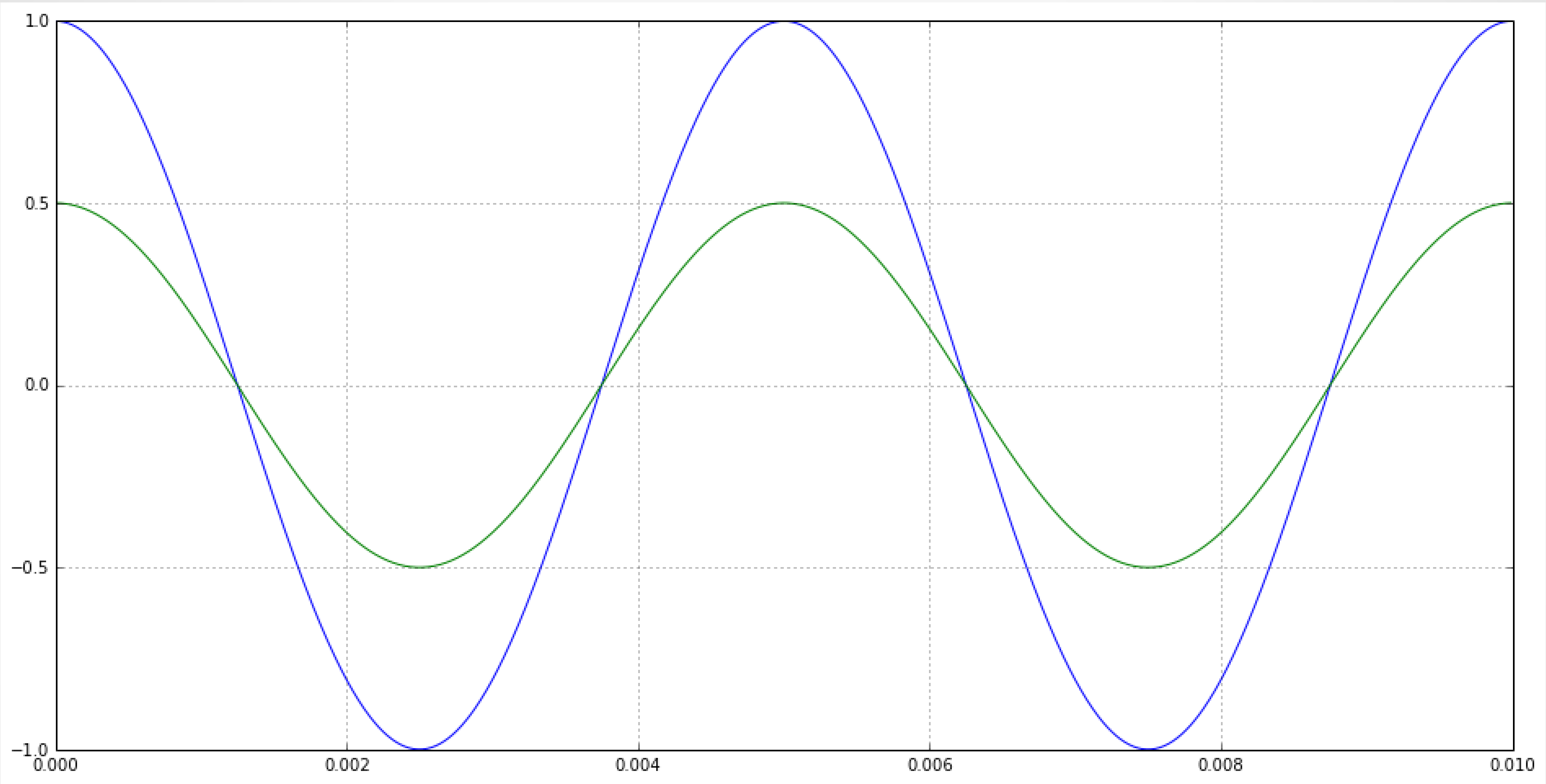
All about altering the properties of a sinusoid.



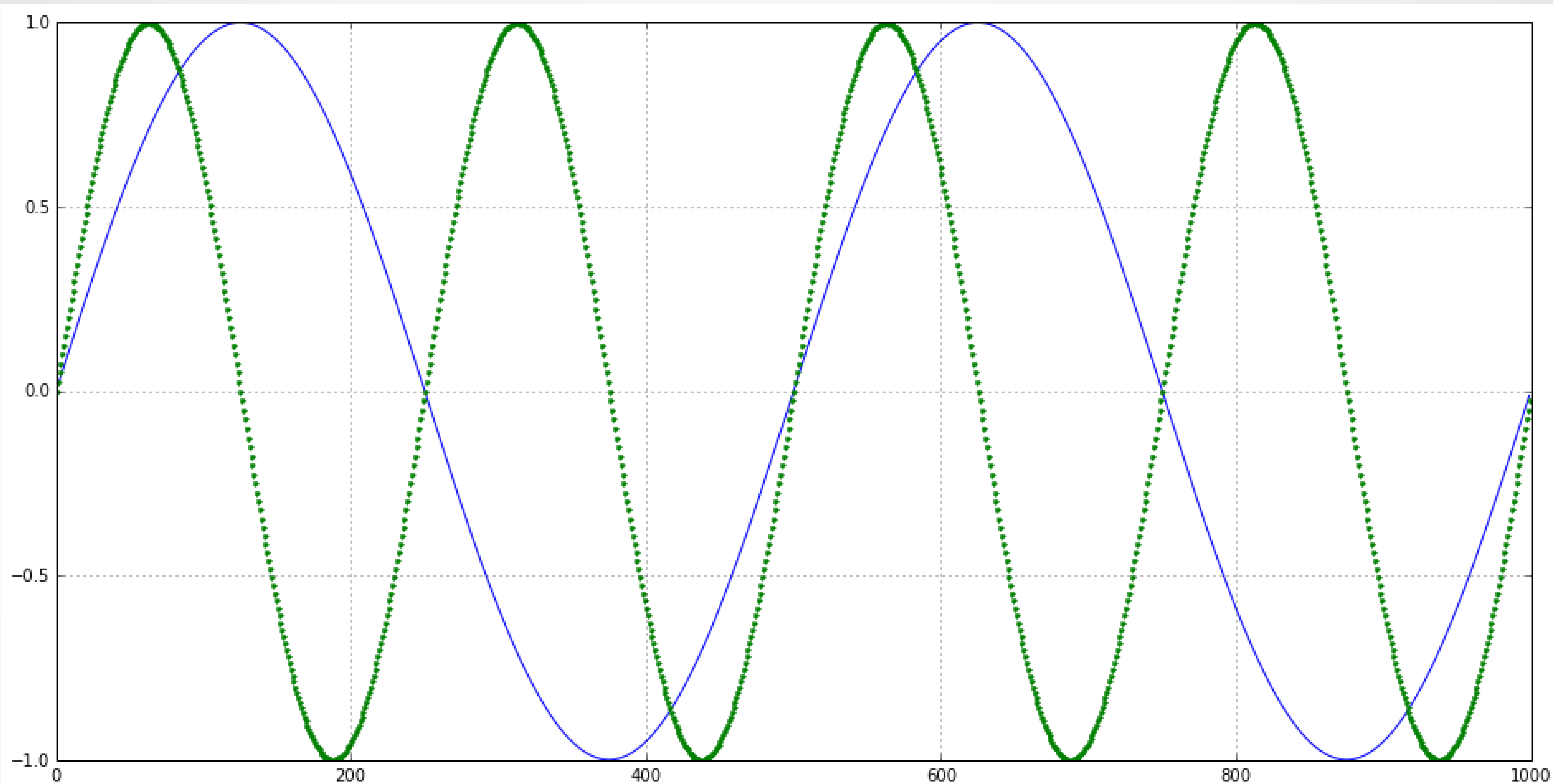
"Carrier Wave"



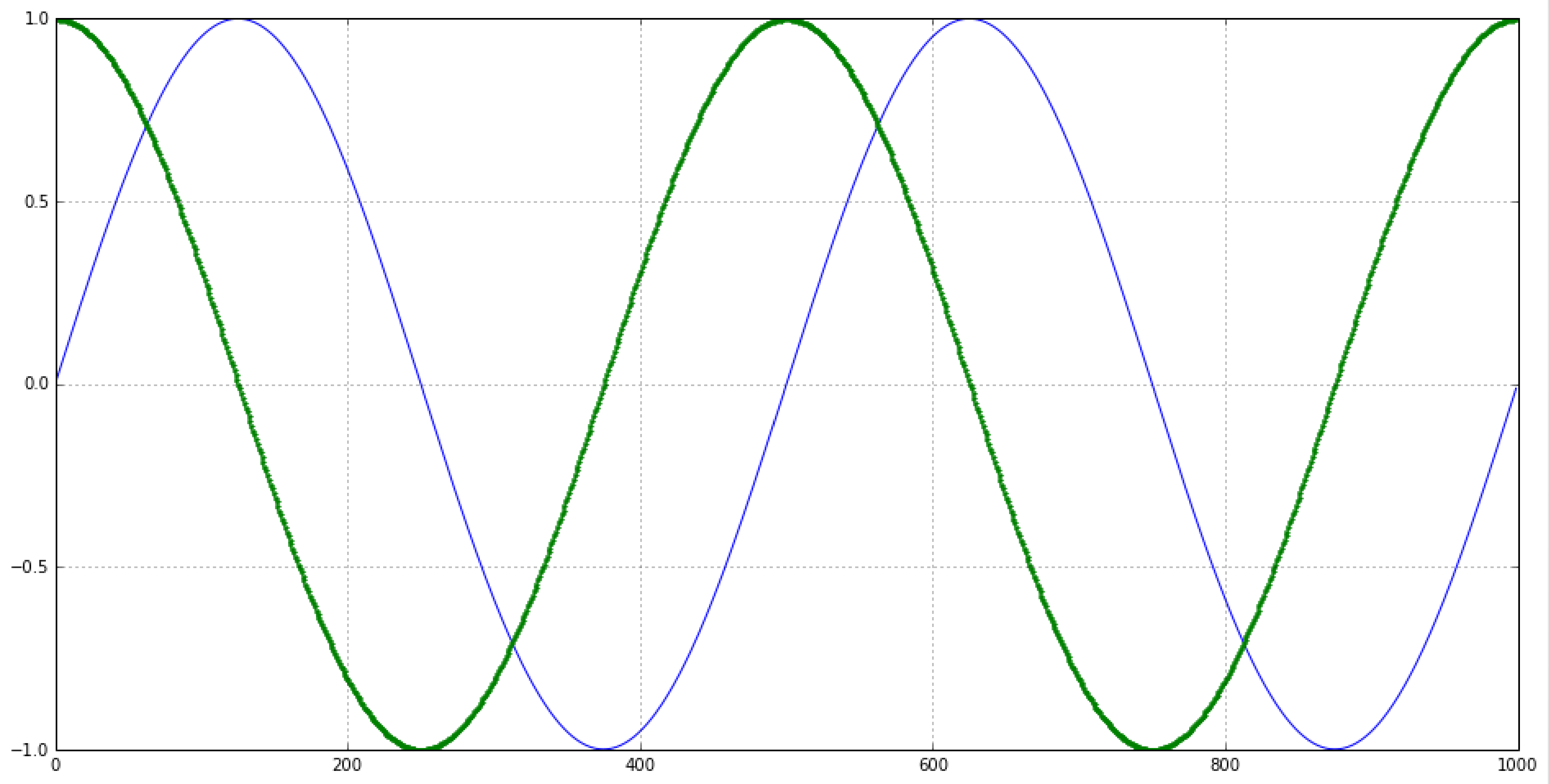
Amplitude



Frequency

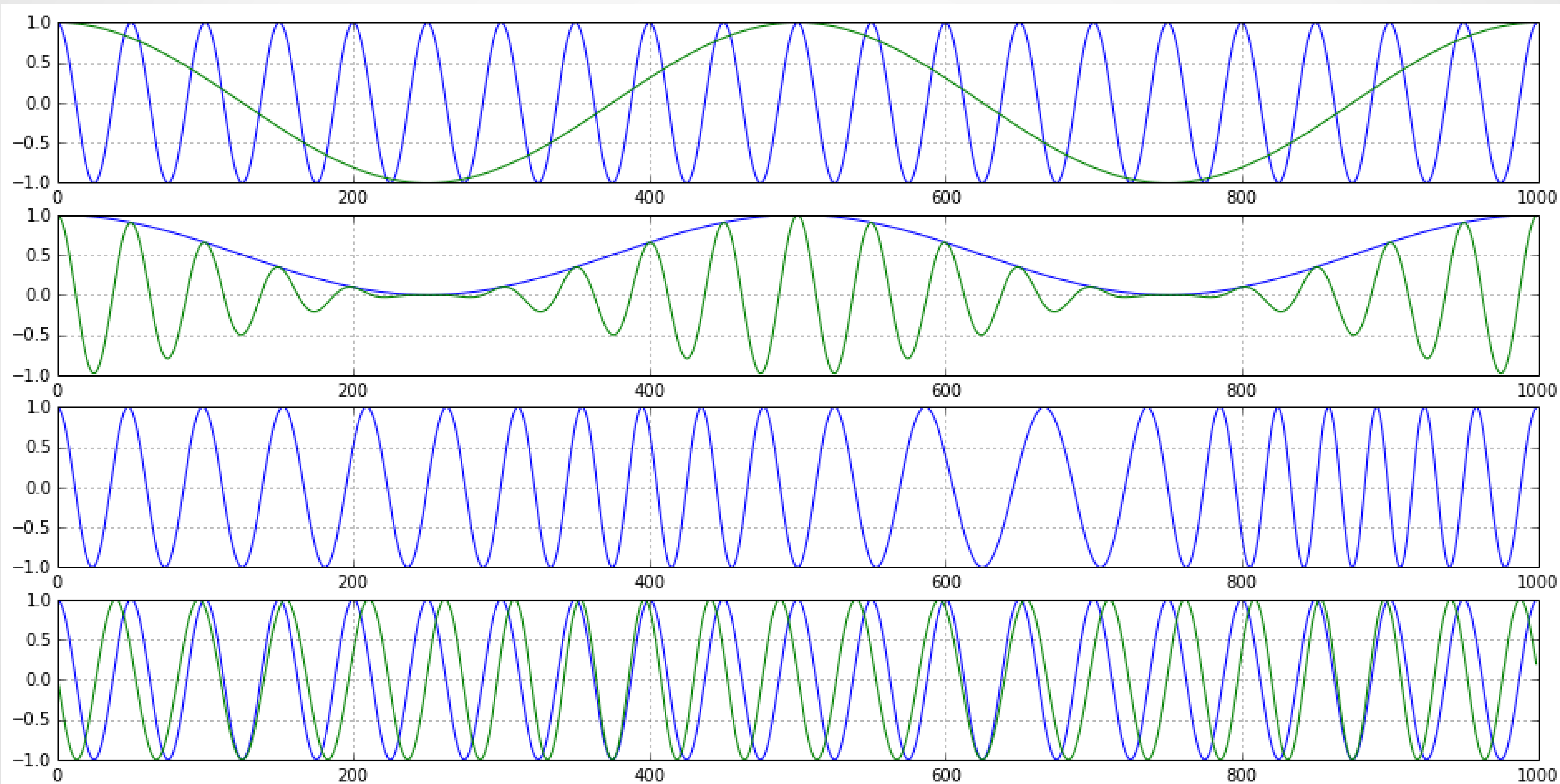


Phase



Modulation - Basic Types

Carrier & Modulation Signal → AM, FM, PM



Modulation - Math

The modulation function:

$$s(t) = a_m(t)\cos((f_c + f_m(t))t + p_m(t))$$

Amplitude Frequency Phase

→ “something(t)” means it <u>may</u> change with time

More concisely:

$$s = a_m \cos((f_c + f_m)t + p_m)$$

AM Modulation

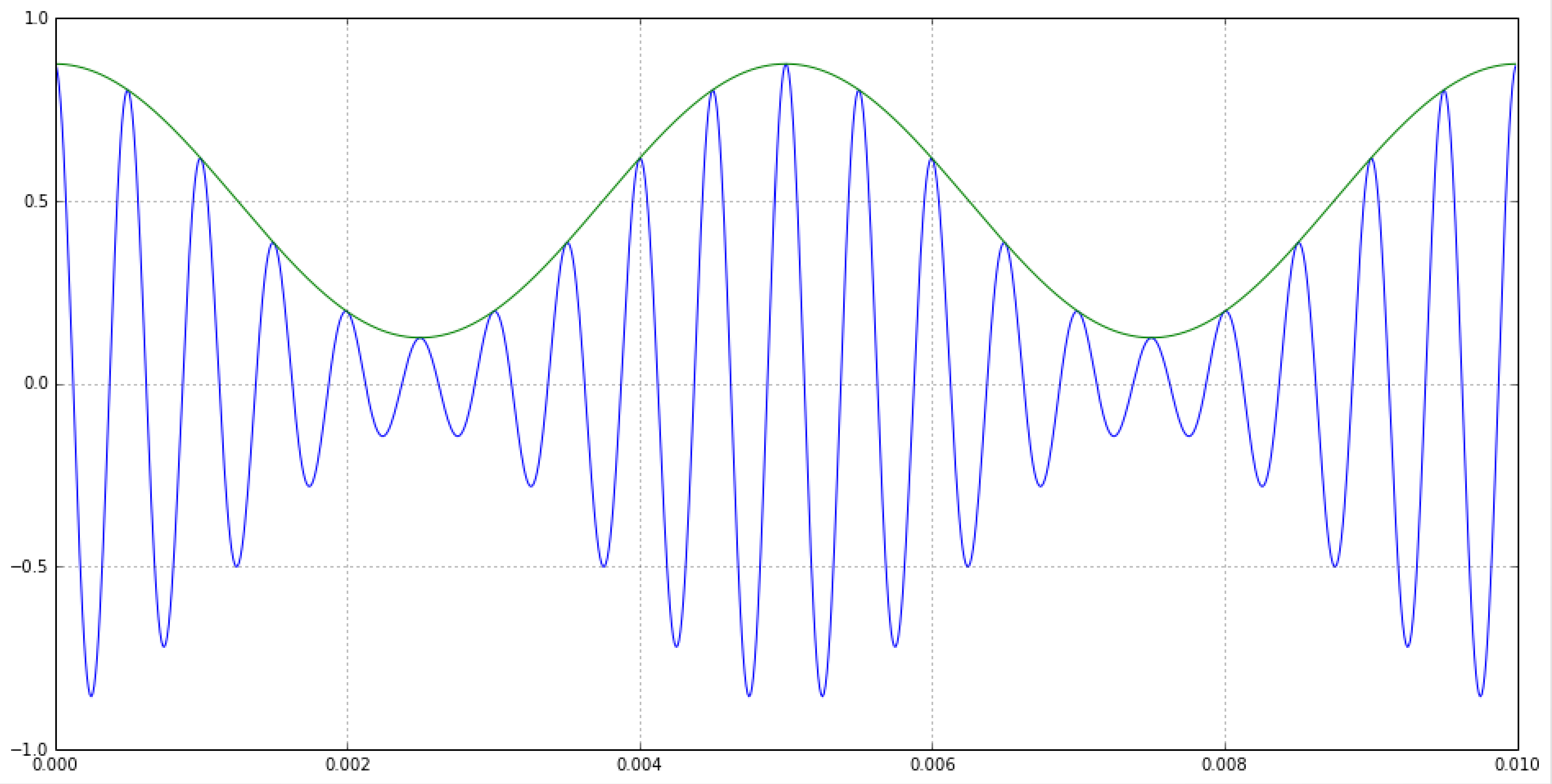
The “universal” modulation function:

$$s = a_m \cos\left(\left(f_c + f_m\right)t + p_m\right)$$

We are only interested in modulating amplitude, so the frequency and phase components drop out.

$$s_{am} = a_m \cos\left(f_c t\right)$$

AM Waveform



AM - Math

Recall the AM function:

$$v_{am} = v_m \cos(\omega_c t)$$

The modulation signal v_m is in the range 0 to 1, where 0 yields zero signal output, and 1 yields 100% carrier amplitude.

We want to test our modulation with a sinusoid input so we need to scale and shift it so it is in the range 0 to 1.

$$v_m = \frac{1}{2}(\cos(\omega_m t) + 1)$$

AM – Math 2

$$v_{am} = v_m \cos(\omega_c t) \qquad v_m = \frac{1}{2}(\cos(\omega_m t) + 1)$$

$$v_{am} = \frac{1}{2}(\cos(\omega_m t) + 1)\cos(\omega_c t)$$

$$v_{am} = \frac{1}{2}\cos(\omega_m t)\cos(\omega_c t) + \frac{1}{2}\cos(\omega_c t)$$

AM – Math 3

$$v_{am} = \frac{1}{2}\cos(\omega_m)\cos(\omega_c) + \frac{1}{2}\cos(\omega_c)$$

Use a trigonometric identity to separate the cosine product:

$$\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

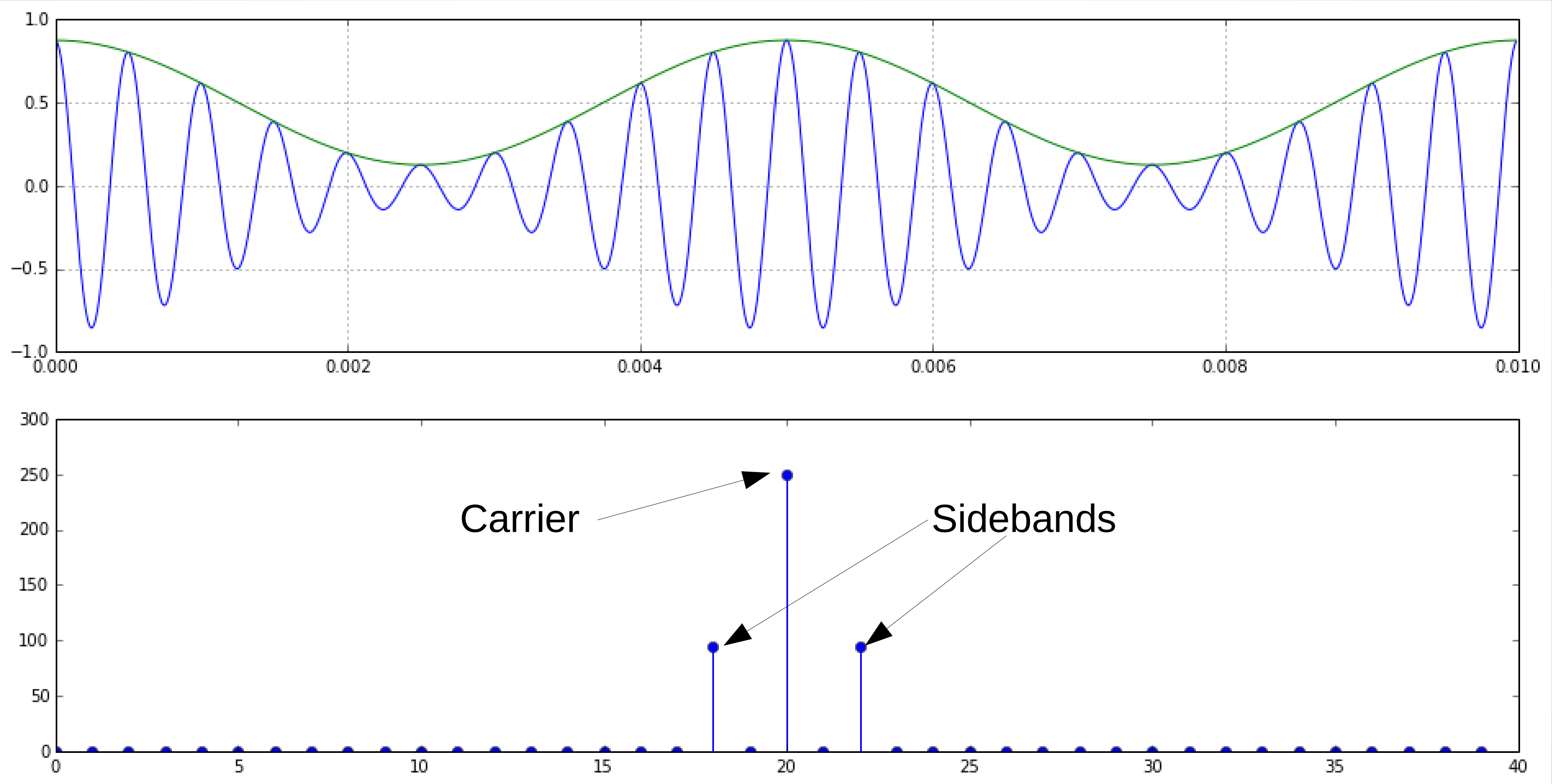
$$v_{am} = \frac{1}{4}\cos(\omega_c - \omega_m) + \frac{1}{4}\cos(\omega_c + \omega_m) + \frac{1}{2}\cos(\omega_c)$$

Lower Sideband

Upper Sideband

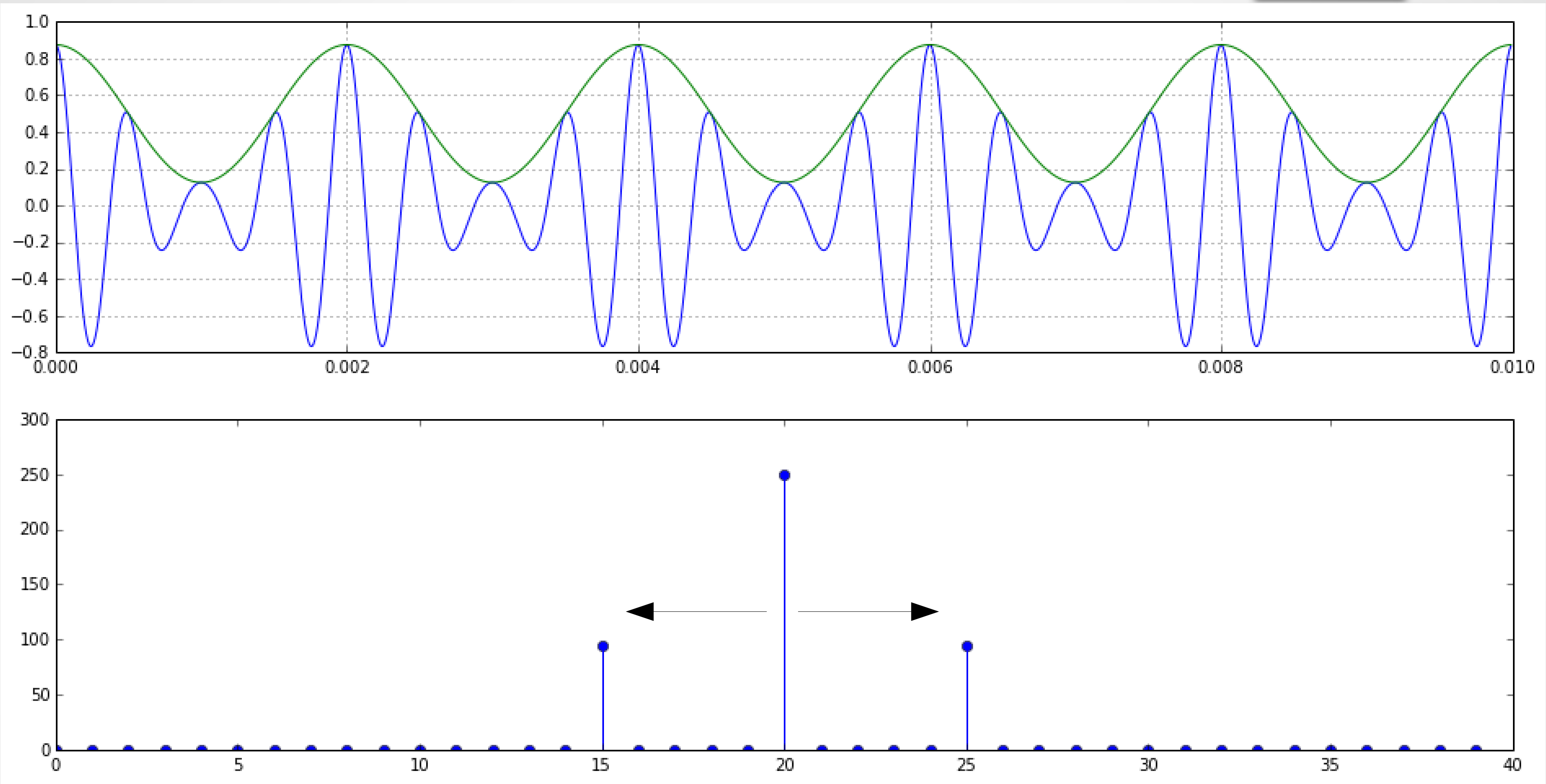
Carrier Component

AM - Spectrum



1.0
0.5
0.0
−0.5
−1.0
0.000
0.002
0.004
0.006
0.008
0.010
300
250
200
150
100
50
0
0
5
10
15
20
25
30
35
40
Carrier
Sidebands

AM – Spectrum 2



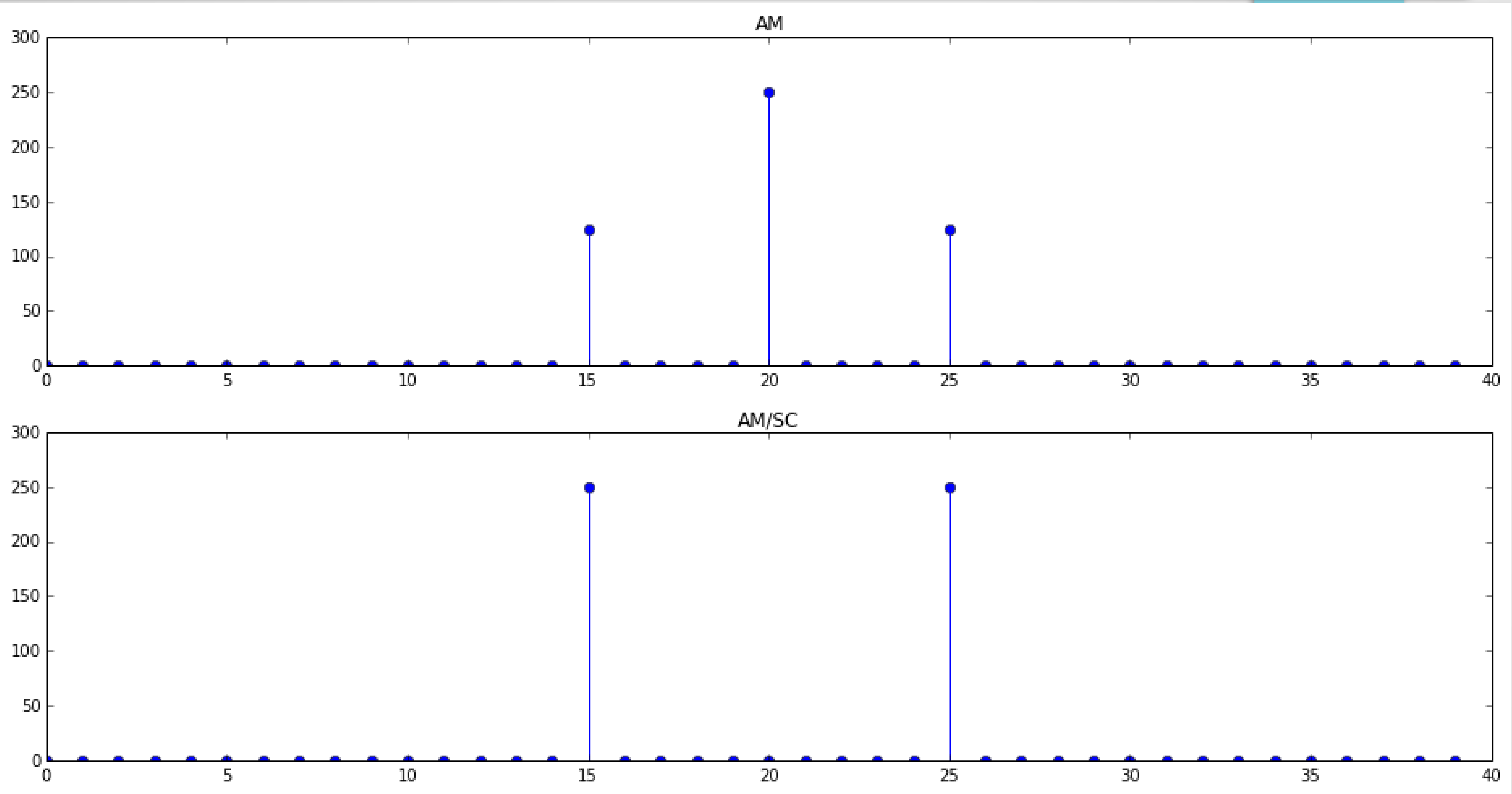
Increased modulation signal frequency increases distance from carrier.

Inching Toward SSB – AM/SC

AM wastes a lot of energy in the carrier component, can we fix that?

AM with Suppressed Carrier (SC)

AM/SC - Spectrum



That is what it looks like, but how can we make it?

AM/SC - Math

Recall the AM function: $$v_{am} = v_m \cos(\omega_c t)$$

For AM, v_m was in the range 0 to 1.

For SC, make the range +/- 1, just a regular sinusoid.

$$v_m = \cos(\omega_m t)$$

AM/SC – Math 2

$$v_{am} = v_m \cos(\omega_c t) \qquad v_m = \cos(\omega_m t)$$

$$v_{am} = \cos(\omega_m t)\cos(\omega_c t)$$

$$\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

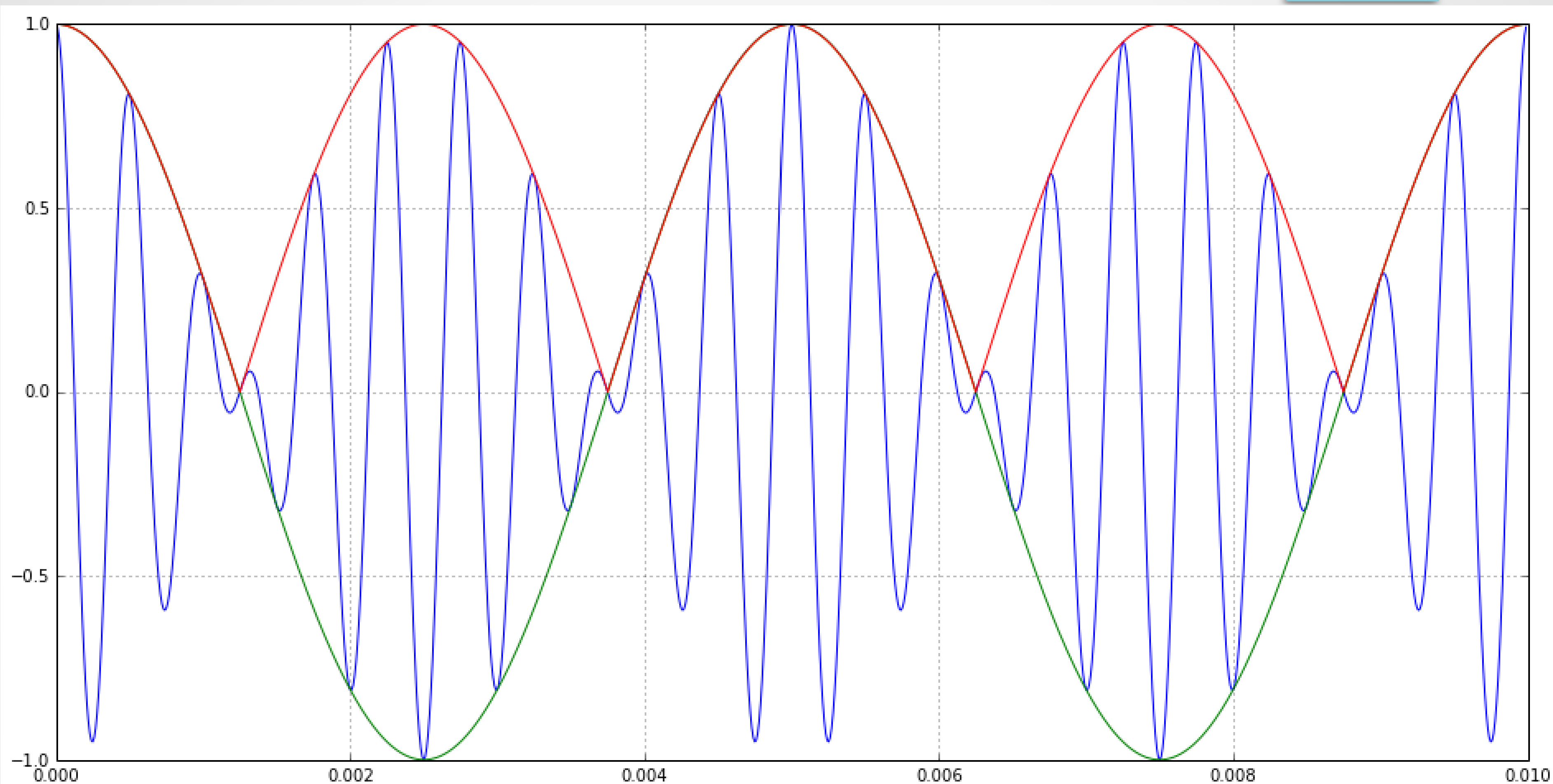
$$v_{am} = \frac{1}{2}\cos(\omega_c - \omega_m) + \frac{1}{2}\cos(\omega_c + \omega_m)$$

Lower Sideband

Upper Sideband

No Carrier Component!

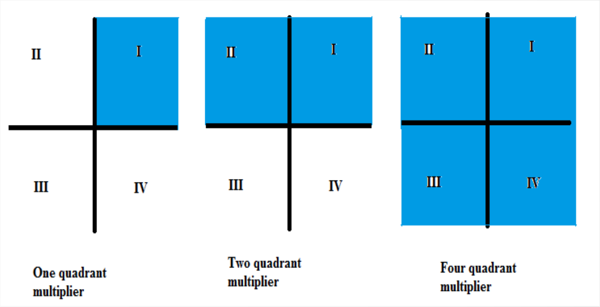
AM/SC “breaks” AM

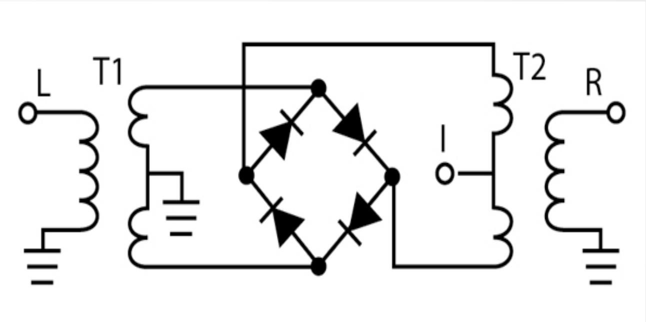


Wrong envelope! Carrier amplitude inversion!

SC/SSB – Electronics

- Multiplying negative values
  - One, two, and four quadrant
- Switchers (Mixers)
  - Diode Ring (Balanced Mixer)
  - Transistor
- Modulators (Multipliers)
  - Logarithmic amplifiers (Gilbert Cell)





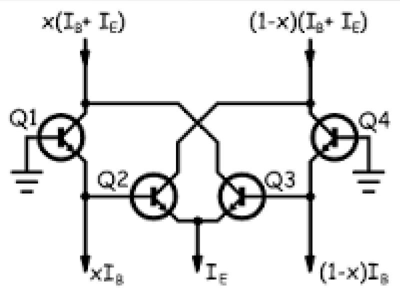
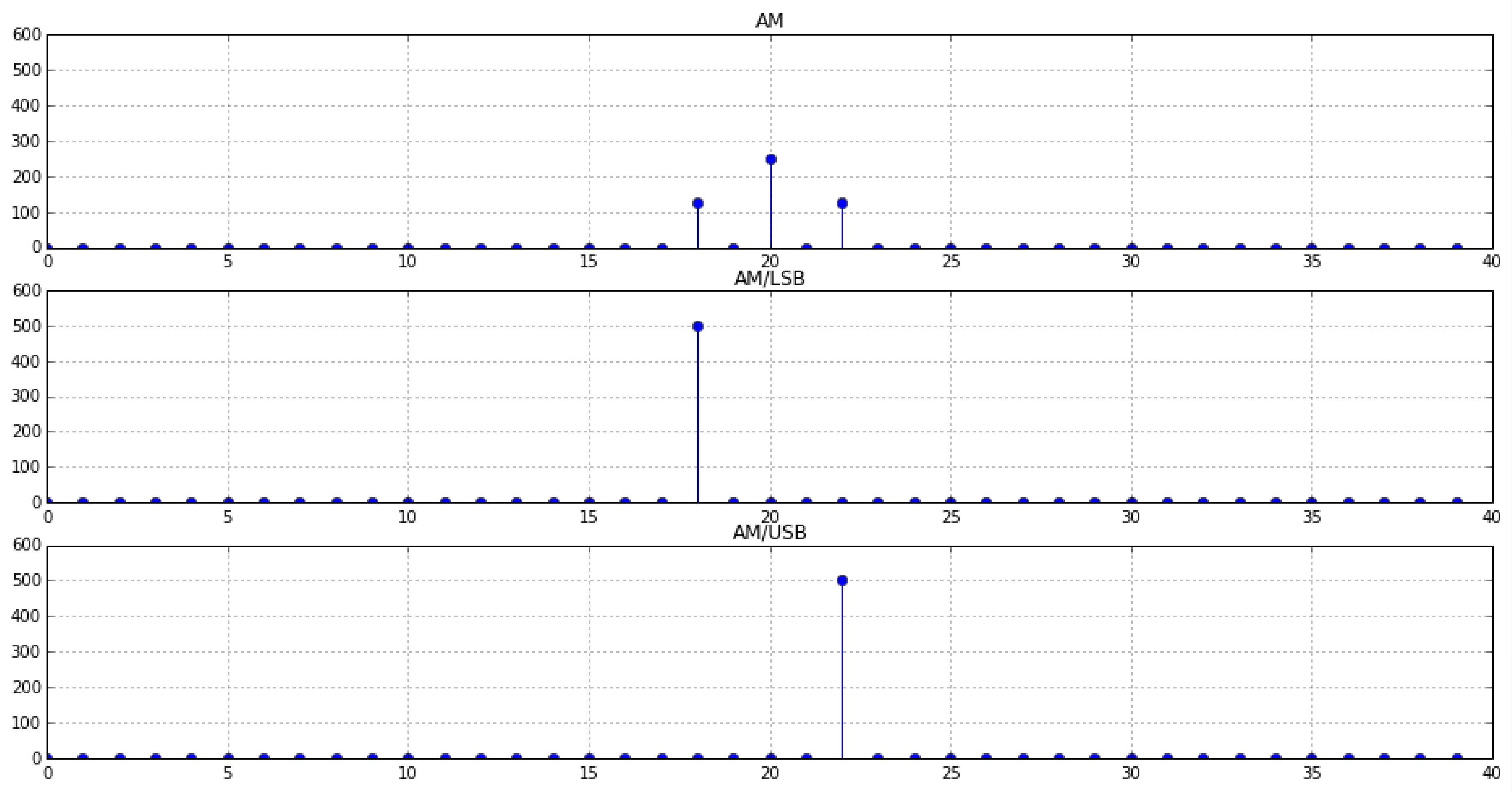


Fig. 1. The Gilbert Gain Cell.

Finally... AM/SSB



That is what it looks like, but how can we make it?

AM/SSB - Math

Remember the cosine product trig identity?

$$\cos(A)\cos(B)=\frac{1}{2}\cos(A-B)+\frac{1}{2}\cos(A+B)$$

Here is another:

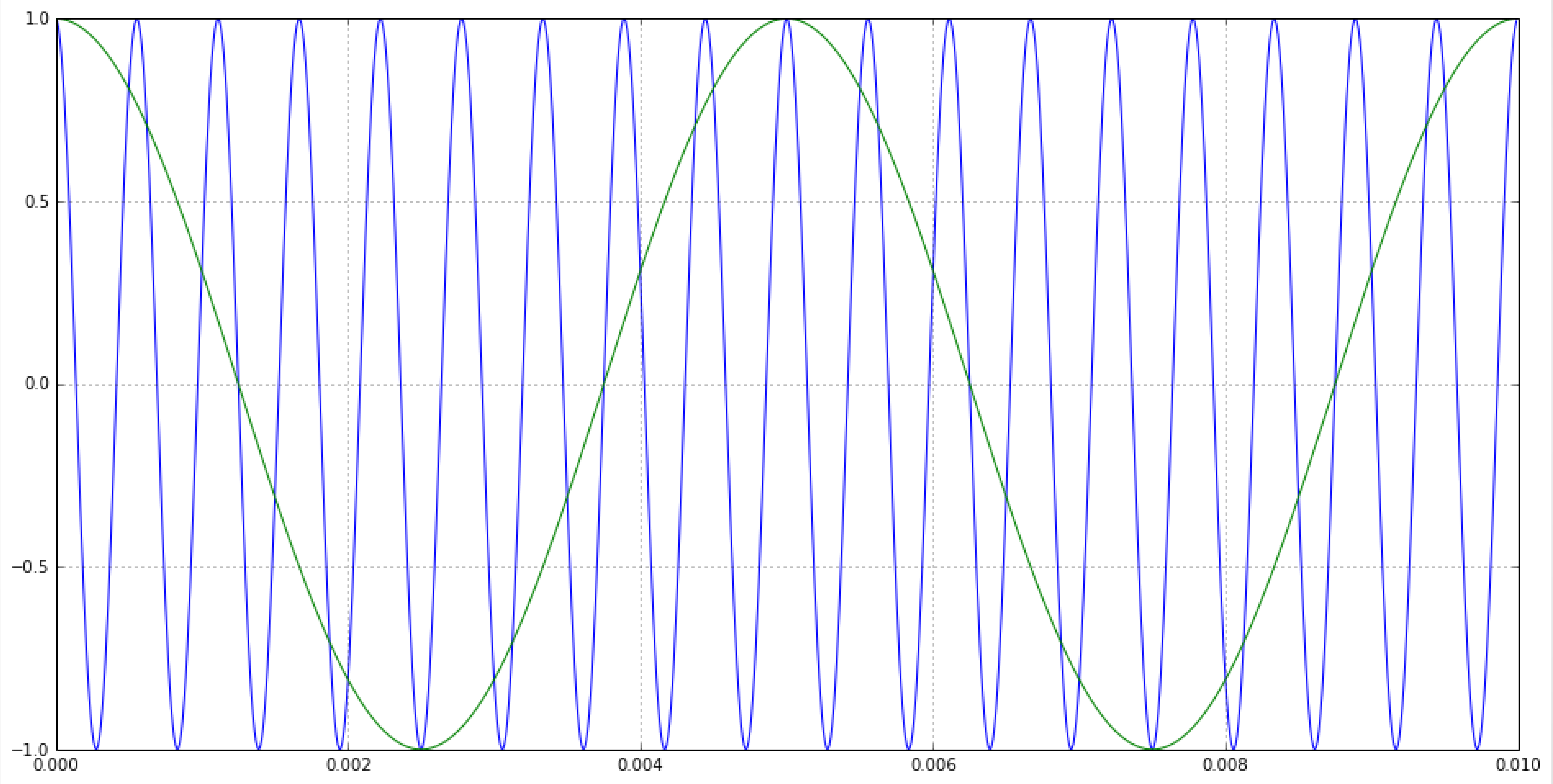
$$\sin(A)\sin(B)=\frac{1}{2}\cos(A-B)-\frac{1}{2}\cos(A+B)$$

Notice that minus sign?

$$\cos(A)\cos(B)+\sin(A)\sin(B)=\cos(A-B)$$

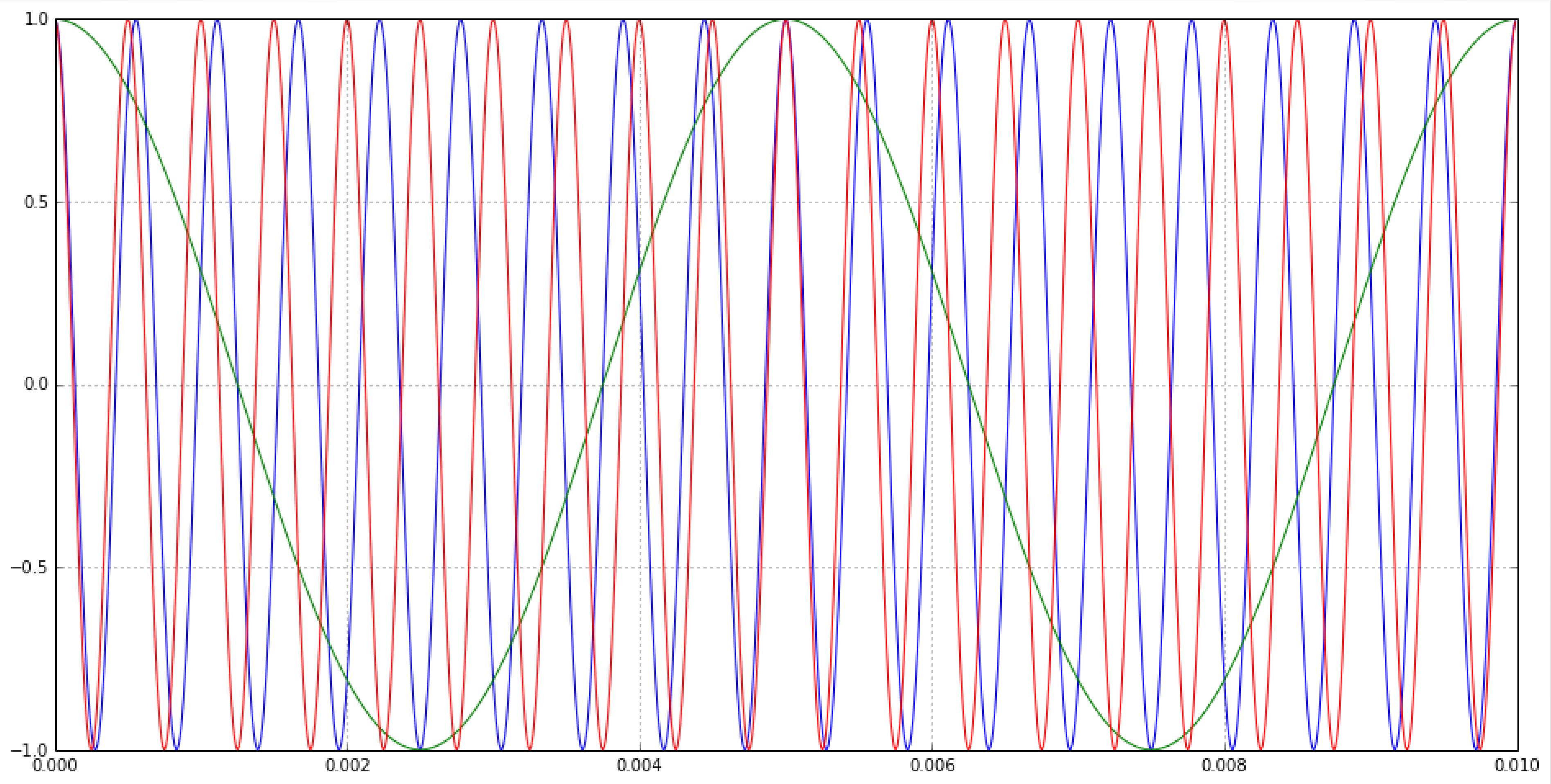
Just the lower sideband!

SSB – Time Domain



Just a sine wave?

SSB – Time Domain 2



Carrier and SSB signal

AM/SSB – Implementation

Simple output, but complicated input...

$$\cos(A)\cos(B)+\sin(A)\sin(B)=\cos(A-B)$$

Recall that sin(x) = cos(x - 90°), so we just need a 90° phase shift of our carrier and modulating signal frequencies.

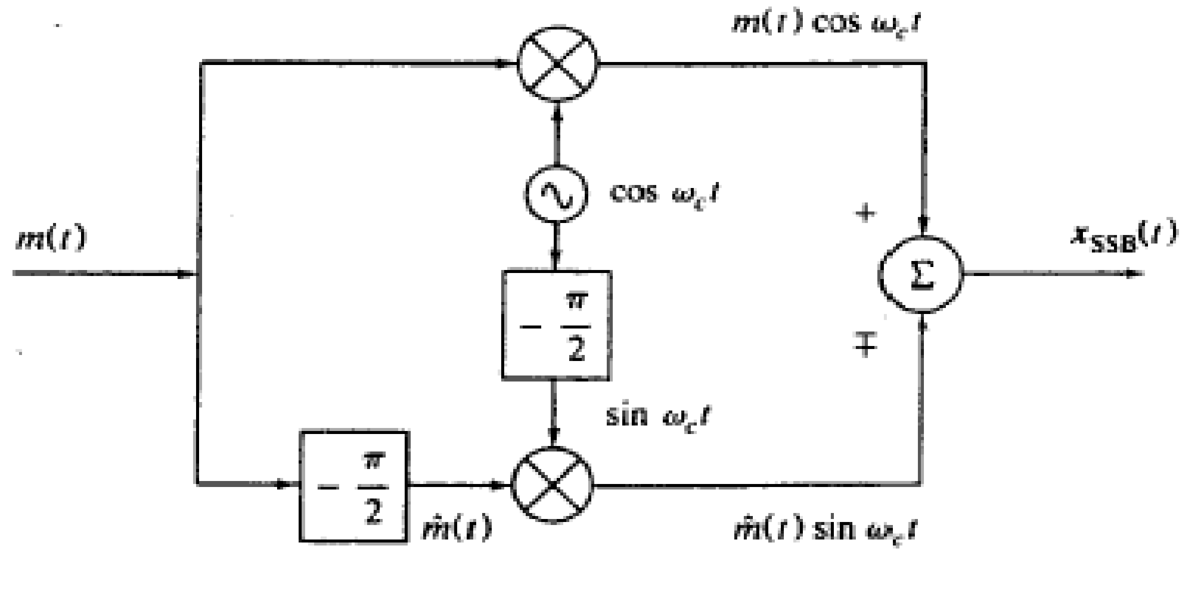
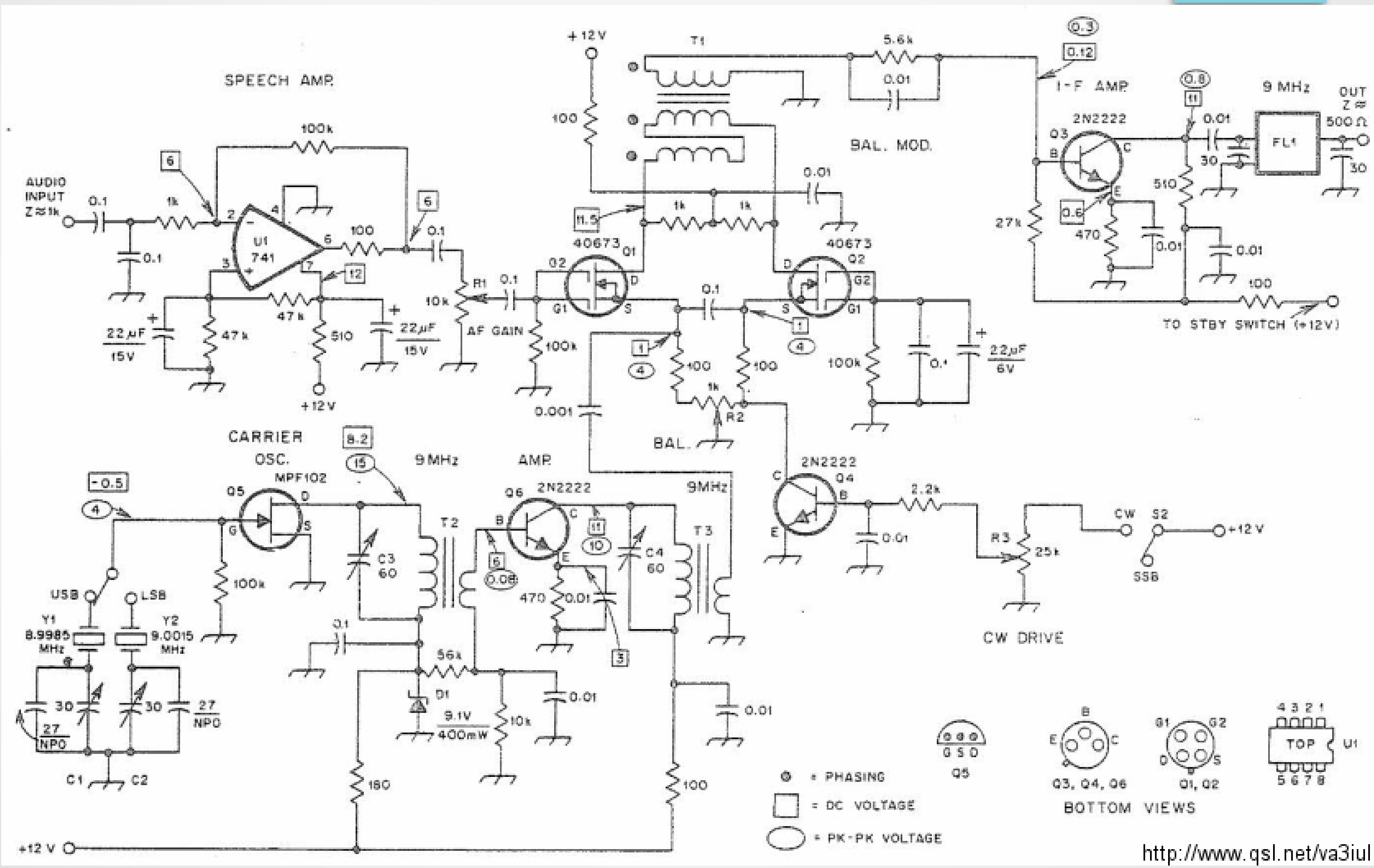


Fig. 3-8 SSB generation using phase shifters

Electronic SSB Modulator, Filtering



This Might Look Familiar...

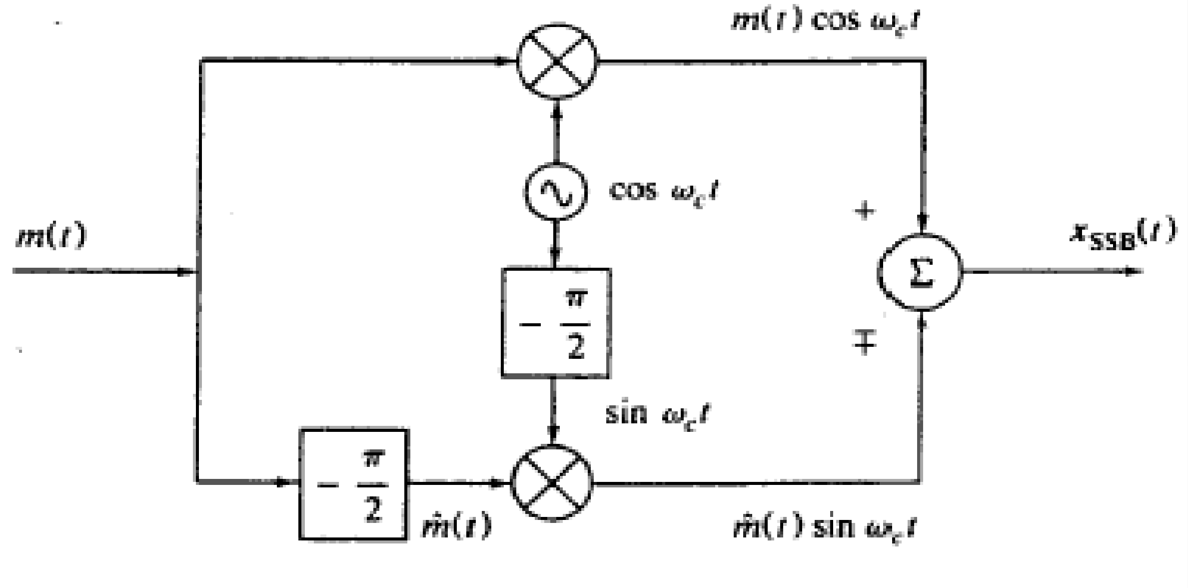
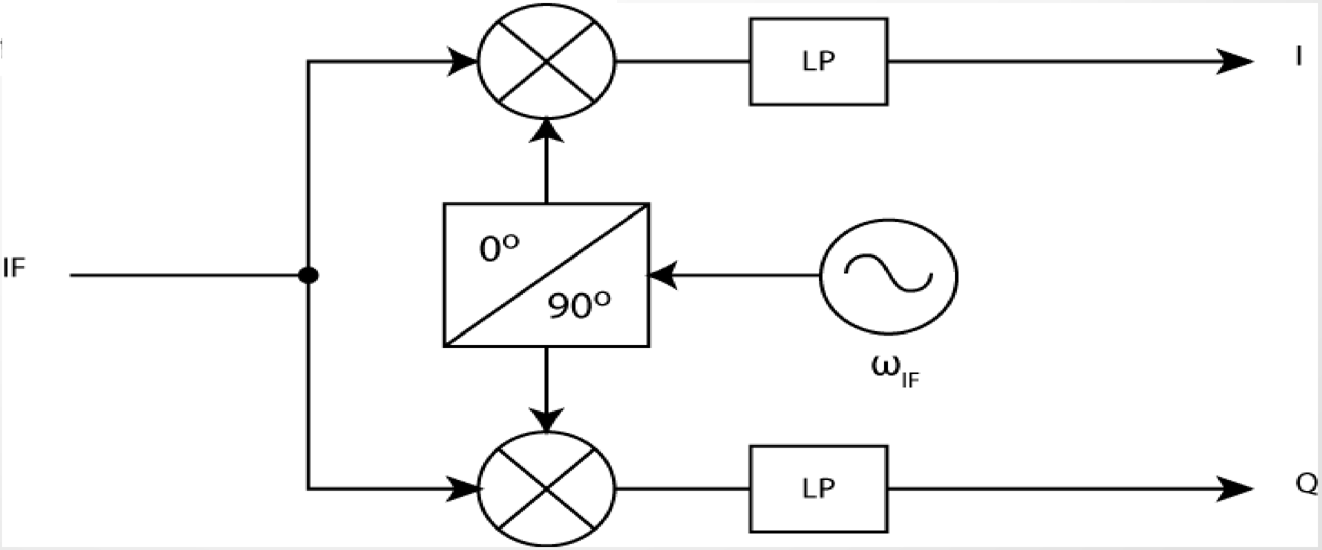
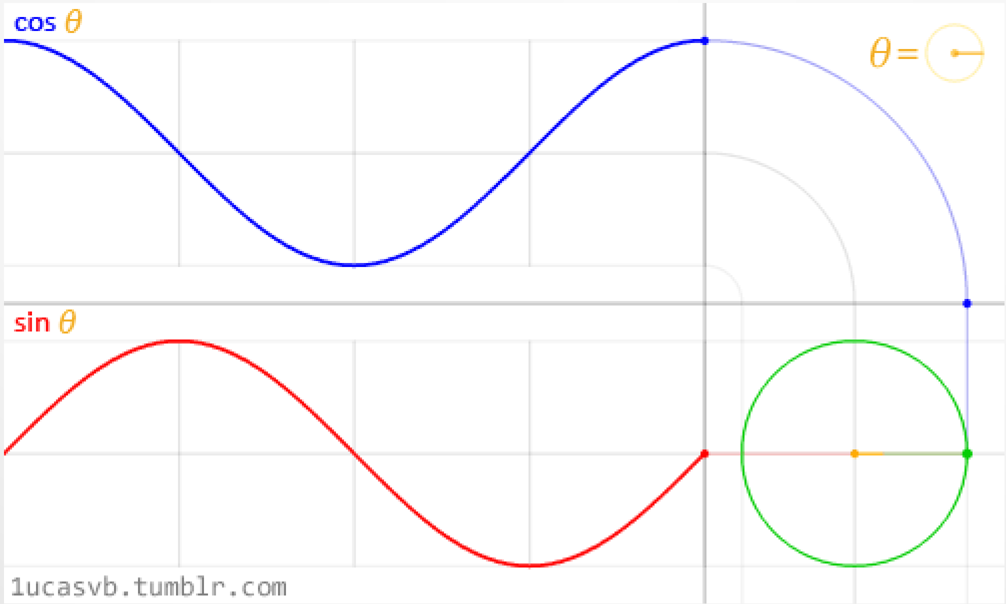


Fig. 3-8 SSB genera



Phase Quadrature

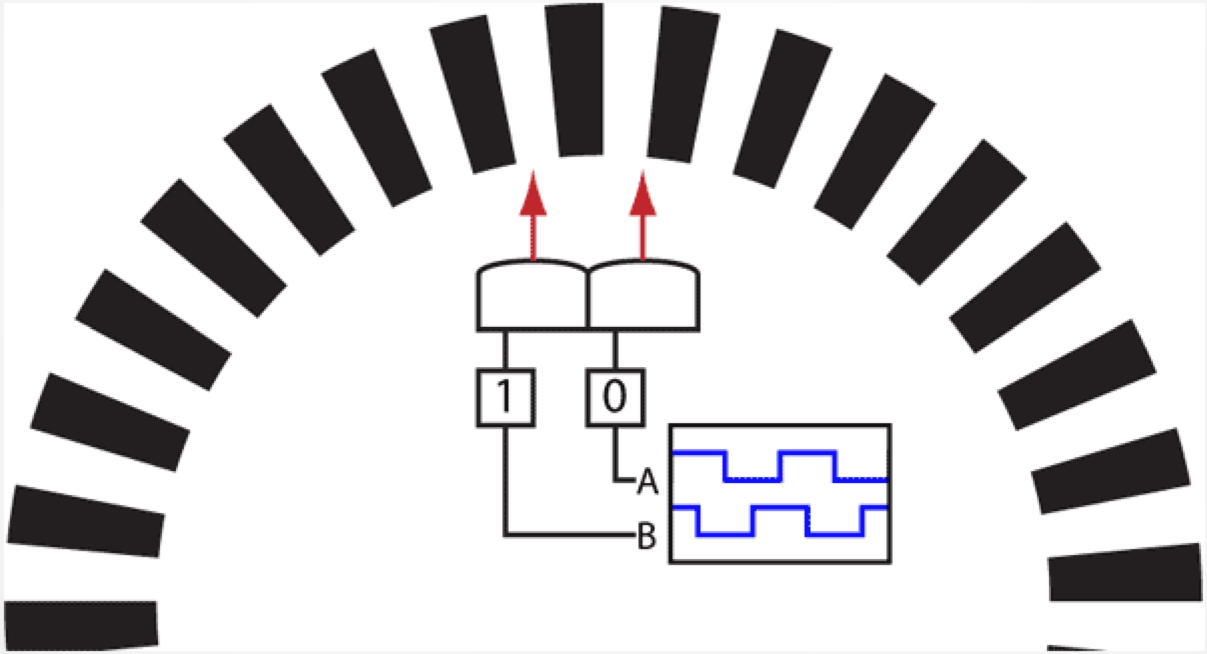
- Quadrature means 90 degrees
  - Latin “Quadratura” - (making) a square
- Sine and Cosine are in quadrature



Quadrature – So what?

- Rotate a point around the unit circle
- Look at either the sine or cosine graph
  - Can you tell rate of rotation (frequency)?
    - Yes.
  - Can you tell the direction of the rotation?
    - No! (But you can if you see both sine and cosine)
- So quadrature allows frequency <u>and</u> direction
- E.g. “Negative Frequency”

Quadrature Sensors

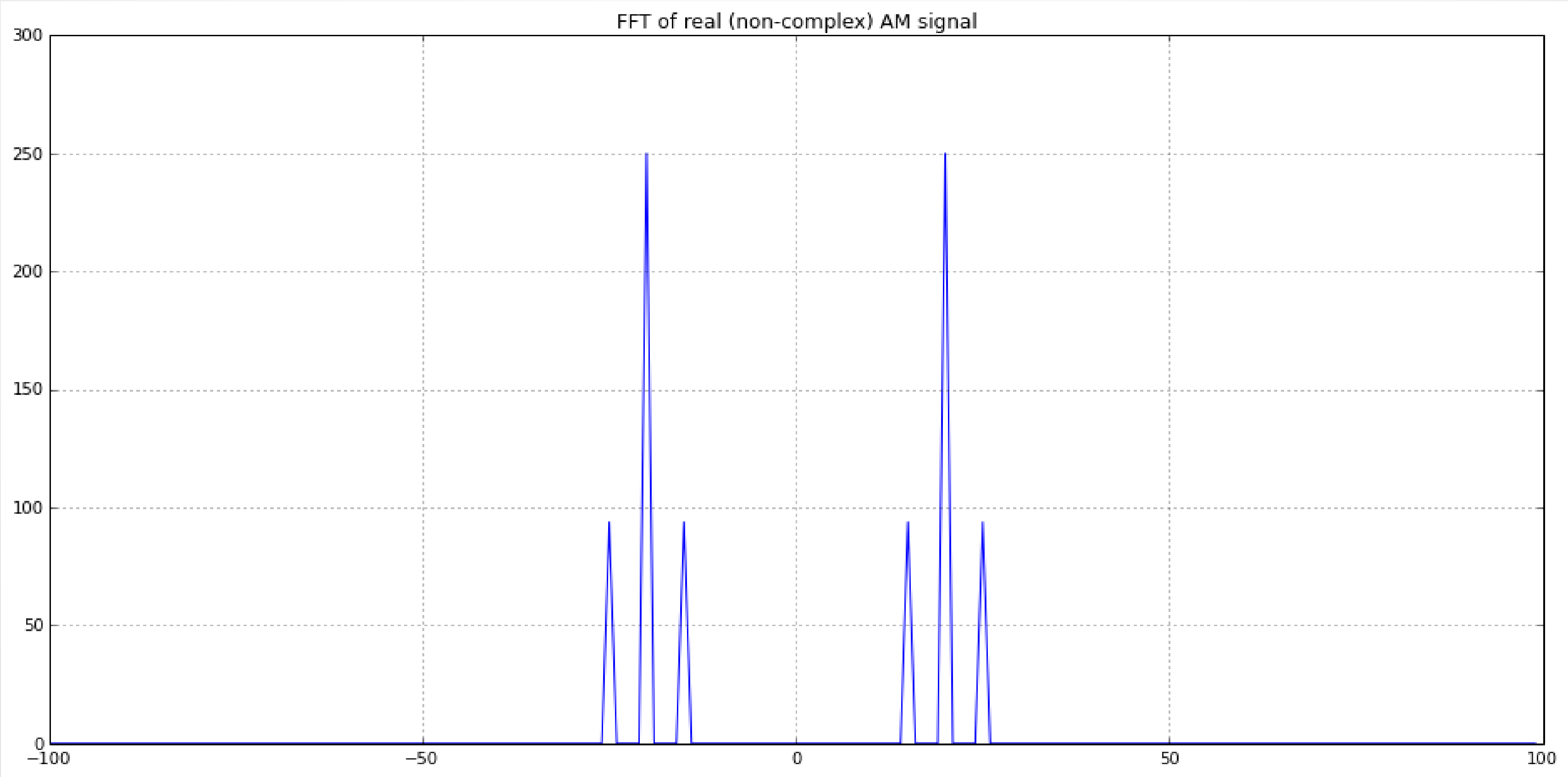


http://www.creative-robotics.com/quadrature-intro

Real Signals

No difference between positive or negative frequencies.

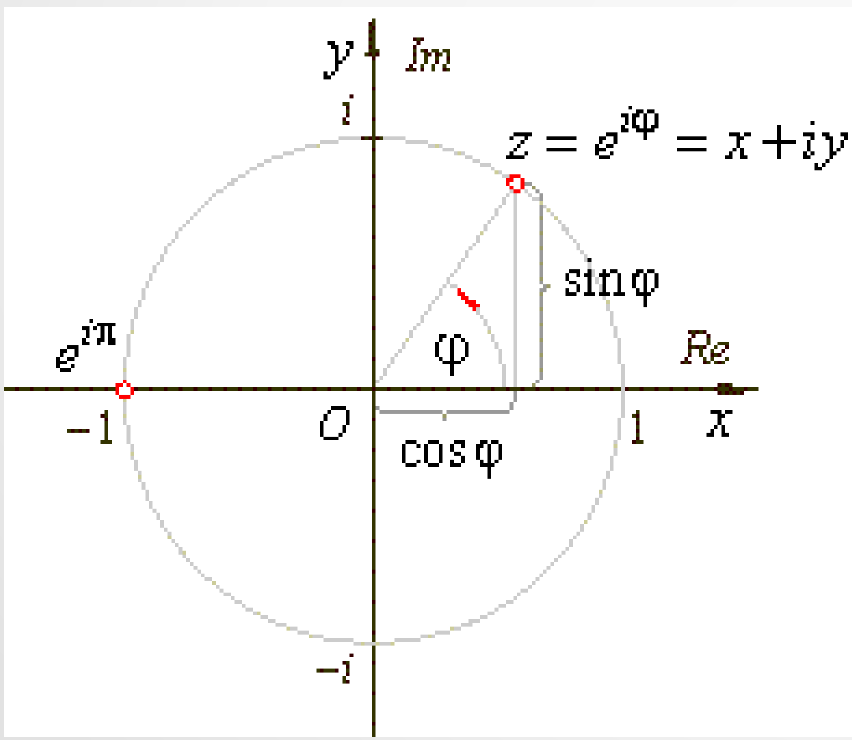
Thus upper and lower sideband mirror images.

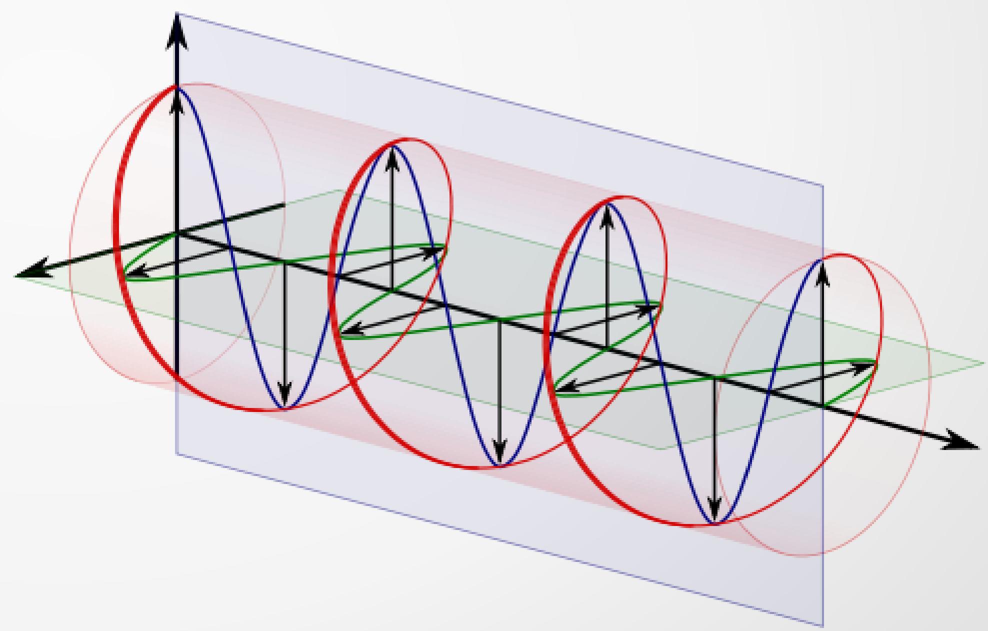


IQ Signals Make You Smarter

I = In-phase → Cosine →Real

Q = Quadrature → Sine → Imaginary





Why Complex Numbers?

- Why not just treat I and Q as two real values?
- In some ways IQ is like 2x sample rate, some ways not...
  - 2x bandwidth, but still no “negative frequency”
  - 90° offset is key, 2x would be 180°
- Complex numbers represent the relationship between I&Q, especially during operations; like multiplication.
- Beautiful Math... Euler's Formula:

$$e^{ix} = \cos x + i \sin x$$

Complex Sinusoid

SSB IQ

$$v_{ssb} = v_m e^{i\omega_c t}$$

SSB is just a multiplication in the complex domain!

(As is any frequency translation)

Radio Selection

- Transmit? Full duplex?
- Frequency Range
- Sampling Rate / Bandwidth
- ADC resolution
- On-board DSP – FPGA, CPU
- Connectivity
  - USB2/3, Ethernet, PCIe, …
-

Some Radios...

- Realtek Dongles
- HackRF
- Ettus Research
- BladeRF
- LimeSDR

Application Ideas

- Basic AM/FM modulation
  - Multi-channel relay
- Packet Radio
- Satellite
- Direction Finding
- RADAR
- Atmospheric/propagation monitoring, Ionosphere, etc.
- HAM IoT

Demos

- GNU Radio
- Fosphor
  - Tx spectrogram image
- Simple AM/SSB/FM radio, CTCSS, trunking
  - A CTCSS multi-channel full duplex relay
- Digital Modes – OFDM
- Simultaneous Audio, Slow-scan video, data
- Digital audio